



Magnetometer Calibration Using Ellipsoid Fitting Method

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ARTICLE INFO	ABSTRACT
<p>Article History: Received 25 November 2018 Received in revised form 4 January 2019 Accepted 11 March 2019 Available online 12 March 2019</p> <p>Keywords: Calibration Method, Magnetometer, Magnetic Field, Fluxgate</p>	<p>Given the wide range of applications for magnetic sensors, particularly magnetometers, precise calibration is essential to ensure reliable and accurate measurements. Magnetic sensors are highly sensitive to both internal imperfections and external disturbances, including environmental magnetic fields, which can significantly affect their output. To approximate real-world conditions, this study involved mounting a magnetometer on a vehicle and conducting measurements in a remote, low-interference region in Iran. The vehicle was driven within a limited radius to collect data under relatively undisturbed geomagnetic conditions. The raw data acquired by the sensor was recorded and subsequently processed using MATLAB software, employing the Ellipsoid Fitting Method for calibration. This technique is widely recognized for correcting both hard-iron and soft-iron distortions typically present in three-axis magnetometers. The study begins with a concise overview of common magnetometer calibration approaches, comparing their principles and practical implications. It then presents the experimental setup, data collection procedure, calibration process, and outcomes in detail. The results indicate a significant improvement in the accuracy of the magnetometer readings after calibration, demonstrating the effectiveness of the adopted method. This investigation highlights the practical considerations and challenges involved in magnetometer calibration under real-world conditions and contributes to the optimization of sensor performance for future field applications.</p>

1. INTRODUCTION

Precise measurement of the magnetic field is extensively applied in geophysical researches, space missions and satellites, defense and aerospace industries, exploration of mineral reservoirs, drilling and tunneling and even robotic activities. Accurate positioning is one of the most necessary requirements for an extensive range of applications. Living organisms, vehicles and even defense systems all need solutions for satisfying such requirements. Positioning methods could be generally divided into two categories of Active and Passive modes. Active methods require send and receipt of data and a source. For instance, several radio frequency transmitters with defined positions in a region are required so that the system calculates its position based on the received waves from such transmitters and to calculate its distance with each one of them, and or the device itself transmits waves and calculates its own position

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upon receipt of the reflection of its waves, and or in GPS application, the waves are transmitted from the existing satellites and the system calculates its position based on the position of the satellites. However, these systems have some defects too, such as non-coverage at many locations and insecurity and low dependability (for example, these systems can be challenged with transmission of radio disturbances). Also, their accuracy can be pretty low depending on the environment and the frequency of the waves and even other factors.

On the other side, Passive methods do not require an external source, but use existing natural data. Positioning based on inertia, earth gravity and magnetic field are of the most accessible and significant existing natural sources for this purpose. Position of the stars is also one of the solutions which specifically can be used for space applications. The above-mentioned sources are both used as independent sources for positioning and also, in some researches, for correcting the measurement errors using other methods and a combination of them [1].

2. MAGNETOMETERS

Using the earth's magnetic vector for positioning at its first stage require accurate measurement of this vector which can be acquired using magnetic sensors and upon its calibration.

The most common types of magnetometers include Helium Magnetometers with Optical Pumping, Proton Magnetometers, Three-Axis Fluxgate Magnetometers and Anisotropic Magneto-resistance Magnetometers. The first two types can only calculate the magnetic field amplitude, while the other two which are categorized as vector magnetometers are actually of the most popular sensors in AHRS navigation systems and their significant feature is their low price and high dependability [2].

Figure 1 is a schematic diagram of combining single-axis ring type sensors which demonstrate formation of a three-axis fluxgate sensor [3].

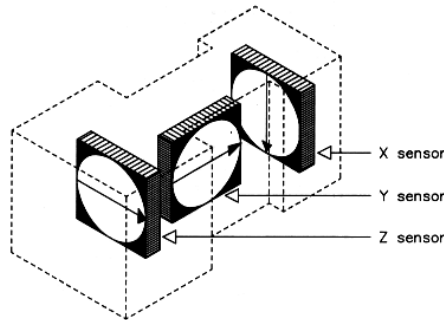


Fig. 1. Schematic diagram of fluxgate sensor

However, there are various error factors associated with the measurement of such sensors. In general, the sources causing measurement error for such sensors can be divided into three generalized categories [4]:

Errors in the magnetometer's sensors due to defects in the production process which include sensitivity error and zero deviation.

Errors derived from installation and nonalignment of the magnetometer with the main axes of the devices and other sensors which include non-orthogonal error (sensor's axes are not perpendicular to each other) and nonalignment error.

External errors derived from magnetic disturbances and measurement noise, which include soft and hard magnetic materials and magnetic field generators such as current carrying wires and errors in measurements of electronic circuits [5].

Firstly, we address introduction and separation of different aspects of calibration process [5].

2.1. Sensor's frame calibration process

Each and every sensor should be correctly calibrated in terms of compensating the sensitivity, offset (zero deviation) and axes non perpendicularity errors. After this stage, we can account an equivalent physical value for the output voltage of the sensor. This stage can be done once at the manufacturing factory, however it is appropriate to conduct such calibration could also be repeated periodically, as the sensitivity of the sensors and their offset value can change with time and also due to other factors such as temperature. Even sometimes it is necessary to calibrate the device every time it is restarted [6].

2.2. Sensor frame installation calibration process

In order to precise acquisition of the quantities, we need to accurately determine the geometric rasion between axes of different sensors. Conducting this measurement once is sufficient and can be performed at the factory or prior the first use after being installed on the device [7].

2.3. Anatomic calibration process

At the last stage, we should account for and compensate the ratio between the sensor and a part of the body which it is mounted on. As a matter of fact, the aim to use the sensor is to determine the direction of the body of the object that it is installed on, and therefore in order to reach this goal we need to acquire this value too. This stage shall be performed every time that the sensor is reinstalled for various applications. This stage, of course, can be considered as a part of the previous stage, which are both considered together in these explanations.

3. CALIBRATION METHODS

Almost all of the calibration methods rely on collecting datasets and initial measurements of the sensor. In these methods the sensor is rotating in a magnetic field through various means such as using precise robotic arms or manually or randomly, and or we generate and artificial three dimensional magnetic field around it and the fired is varied in a way that it is like the sensor is actually rotating inside the field. Data acquired from this stage are the basis for further calculations in order to acquire the values of error parameters [8-9].

3.1. Numeric Calibration

After collecting a dataset of measurements which is acquired by the sensor in different directions, a constant principle in such sensor is used for calibration, which is, amplitude of the magnetic field must be constant in all directions and of course equal to the earth's magnetic field which its value is specified considering the placement position of the sensor toward the earth (i.e. height and geographic position) and the ratio of this value should be one, after normalizing the data. The value for magnetic field amplitude equals to the size of the vector derived from vector summation of the output values of the sensors for all three coordinates [10]. This condition can be mathematically expressed as following: [5] and [11].

$$(x_n - o)^T A_e (x_n - o) = 1 \quad \forall_n = 1 \dots N \quad (1)$$

Where, x_n vector is a vector that includes three components of the sensor's measurements output aligned with the three coordinate axes, and N is the number of the data collected in that stage. Matrix O also represents Bias value or zero deviation. Matrix A_e is a symmetric positive and definite matrix which based on that we acquire the correction or data calibration matrix. The major point in this calibration method is that the above equation is actually the equation of an ellipsoid. Therefore this method is an equivalent of a coupling problem for (finding) an ellipsoid. Therefore calibration problem becomes an ellipsoid finding problem which has the most proportionality with the data acquired from the sensor while such data can be placed on its surface with the least rate of error.

3.2. Finding a three-dimensional ellipsoid through nonlinear optimization

Algebraic methods allow us to use the ellipsoid equation in the optimization function and therefore we can provide the cost function as per following. Note that Matrices O and A_e should be acquired in a way that the equation becomes minimum.

$$J(A_e, o) = \sum_{n=1}^N F(A_e, 0; x_n)^2 \quad (2)$$

Where function F and A_e are:

$$F(\overline{A_e}, c; x) = (x - c)^T \overline{A_e} (x - c) - 1 = 0 \quad (3)$$

$$A_e = A^T A \quad (4)$$

Where, A is the calibration matrix. Different methods, such as Newton constrained optimization, have been used in various papers for solving the above equation.

3.3. Finding a three-dimensional ellipsoid through linear optimization

For this purpose, we can rewrite the primary equation as following:

$$x^T A_e x - 2o^T A_e x = 1 - o^T A_e o \quad (5)$$

Where

$$A_e = \begin{pmatrix} p_1 & \frac{p_4}{2} & \frac{p_5}{2} \\ \frac{p_4}{2} & p_2 & \frac{p_6}{2} \\ \frac{p_5}{2} & \frac{p_6}{2} & p_3 \end{pmatrix} \quad (6)$$

$$o = -\frac{1}{2} A_e^{-1} \begin{pmatrix} p_7 \\ p_8 \\ p_9 \end{pmatrix} \quad (7)$$

$$p_{10} = o^T A_e o - 1 \quad (8)$$

And is rewritten as:

$$d(x)^T p = 0 \quad (9)$$

Which we will have:

$$d(x) = [x_1^2, x_2^2, x_3^2, x_1 x_2, x_1 x_3, x_2 x_3, x_1, x_2, x_3, 1]^T \quad (10)$$

This relation is for one measurement and when we have N measurements, then the equation will be as:

$$D_p = 0_{N \times 1}$$

Where Matrix D also called the Design Matrix, will be acquired from inserting matrices d under each other which any of them corresponds to a specific data. We can use an efficient iterative algorithm [12]. The following algorithm should be iterated until the conditions and desired accuracy are met:

Solving the equation $D_p^{(k)} = 0_{N \times 1}$ using the Least Squares method for each step and using the acquired values of the previous step as the initial input values in the new step.

Updating and modification of Matrices $A_e(k)$ and $o(k)$ based on the matrices of the acquired parameters from the previous section.

3.4. Ellipsoid Fitting

The above relations can be also revised as per following for a situation where the data of magnetometer is two dimensional:

$$\begin{aligned} \mathbf{x}^T \mathbf{Q} \mathbf{x} &= [\mathbf{x} \ \mathbf{y}] \begin{bmatrix} \mathbf{q}_{11} & \mathbf{q}_{12} \\ \mathbf{q}_{12} & \mathbf{q}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} \\ &= \mathbf{q}_{11} \mathbf{x}^2 + \mathbf{q}_{22} \mathbf{y}^2 + 2\mathbf{q}_{12} \mathbf{x} \mathbf{y} \end{aligned} \quad (11)$$

$$\begin{aligned} -2\mathbf{b}^T \mathbf{Q} \mathbf{x} &= -2[\mathbf{b}_x \ \mathbf{b}_y] \begin{bmatrix} \mathbf{q}_{11} & \mathbf{q}_{12} \\ \mathbf{q}_{12} & \mathbf{q}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix} = -2(\mathbf{q}_{11} \mathbf{b}_x + \mathbf{q}_{12} \mathbf{b}_y) \mathbf{x} - 2(\mathbf{q}_{12} \mathbf{b}_x + \mathbf{q}_{22} \mathbf{b}_y) \mathbf{y} \\ &= -2(\mathbf{p}_4 \mathbf{x} + \mathbf{p}_5 \mathbf{y}) \end{aligned}$$

$$\|\mathbf{B}_h\|_{N \times 1}^2 = \mathbf{D}_p \quad (12)$$

$$\begin{bmatrix} \|\mathbf{B}_h\|^2 \\ \vdots \\ \vdots \\ \|\mathbf{B}_h\|^2 \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1^2 & \mathbf{y}_1^2 & 2\mathbf{x}_1 \mathbf{y}_1 & -2\mathbf{x}_1 & -2\mathbf{y}_1 \\ \mathbf{x}_2^2 & \mathbf{y}_2^2 & 2\mathbf{x}_2 \mathbf{y}_2 & -2\mathbf{x}_2 & -2\mathbf{y}_2 \\ \mathbf{x}_3^2 & \mathbf{y}_3^2 & 2\mathbf{x}_3 \mathbf{y}_3 & -2\mathbf{x}_3 & -2\mathbf{y}_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{x}_N^2 & \mathbf{y}_N^2 & 2\mathbf{x}_N \mathbf{y}_N & -2\mathbf{x}_N & -2\mathbf{y}_N \end{bmatrix} \begin{bmatrix} \mathbf{q}_{11} \\ \mathbf{q}_{22} \\ \mathbf{q}_{12} \\ \mathbf{p}_4 \\ \mathbf{p}_5 \end{bmatrix} \quad (13)$$

$$\mathbf{D}^T \|\mathbf{B}_h\|_{N \times 1}^2 = \mathbf{D}^T \mathbf{D}_p \quad (14)$$

$$(\mathbf{D}^T \mathbf{D})^{-1} \mathbf{D}^T \|\mathbf{B}_h\|_{N \times 1}^2 = \mathbf{p} \quad (15)$$

$$(\mathbf{b}^T \mathbf{Q})^T = \mathbf{Q} \mathbf{b} = \mathbf{p}_{4,5} \quad (16)$$

$$\begin{bmatrix} \mathbf{q}_{11} & \mathbf{q}_{12} \\ \mathbf{q}_{12} & \mathbf{q}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{b}_x \\ \mathbf{b}_y \end{bmatrix} = \begin{bmatrix} \mathbf{p}_4 \\ \mathbf{p}_5 \end{bmatrix} \quad (17)$$

$$\mathbf{b} = \mathbf{Q}^{-1} \mathbf{p}_{4,5} \quad (18)$$

$$\|\mathbf{B}_h\|^2 = \|\mathbf{A}_h(\widehat{\mathbf{B}}_h - \mathbf{b}_h)\|^2 \quad (19)$$

The presented method converts finding coefficient matrix \mathbf{A} and Bias Matrix \mathbf{b} to finding a five element Matrix \mathbf{p} . According to the below relation, the size of magnetic vector \mathbf{B} is equal to the multiplication of Matrix \mathbf{D} by Matrix \mathbf{p} . where, \mathbf{p} is an auxiliary parametric matrices that is used for calculation of calibration matrices.

$$\|\mathbf{B}_h\|_{N \times 1}^2 = \mathbf{D}_p \quad (20)$$

$$\begin{bmatrix} \|\mathbf{B}_h\|^2 \\ \vdots \\ \vdots \\ \|\mathbf{B}_h\|^2 \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1^2 & \mathbf{y}_1^2 & 2\mathbf{x}_1 \mathbf{y}_1 & -2\mathbf{x}_1 & -2\mathbf{y}_1 \\ \mathbf{x}_2^2 & \mathbf{y}_2^2 & 2\mathbf{x}_2 \mathbf{y}_2 & -2\mathbf{x}_2 & -2\mathbf{y}_2 \\ \mathbf{x}_3^2 & \mathbf{y}_3^2 & 2\mathbf{x}_3 \mathbf{y}_3 & -2\mathbf{x}_3 & -2\mathbf{y}_3 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{x}_N^2 & \mathbf{y}_N^2 & 2\mathbf{x}_N \mathbf{y}_N & -2\mathbf{x}_N & -2\mathbf{y}_N \end{bmatrix} \begin{bmatrix} \mathbf{q}_{11} \\ \mathbf{q}_{22} \\ \mathbf{q}_{12} \\ \mathbf{p}_4 \\ \mathbf{p}_5 \end{bmatrix} \quad (21)$$

The Matrix \mathbf{D} in the above relation is calculated from the values presented by the sensor and the value of all elements of \mathbf{B} is equal to one. Therefore, the above equation can be solved. However, solving the above equation will be difficult when the number of elements is very large. In order to solve this issue, it was decided to use the Recursive Least Squares algorithm (RLS).

Upon calculation of Matrix \mathbf{p} , we will be able to acquire Matrices \mathbf{A} and \mathbf{b} based on that, and via the following relations:

$$\mathbf{Q} = \mathbf{A}^T \mathbf{A} \quad (22)$$

While Matrix \mathbf{Q} is formed from the \mathbf{q} elements of Matrix \mathbf{p} .

$$(\mathbf{b}^T \mathbf{Q})^T = \mathbf{Q} \mathbf{b} = \mathbf{p}_{4,5} \quad (23)$$

$$\begin{bmatrix} \mathbf{q}_{11} & \mathbf{q}_{12} \\ \mathbf{q}_{12} & \mathbf{q}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{b}_x \\ \mathbf{b}_y \end{bmatrix} = \begin{bmatrix} \mathbf{p}_4 \\ \mathbf{p}_5 \end{bmatrix} \quad (24)$$

$$\mathbf{b} = \mathbf{Q}^{-1} \mathbf{p}_{4,5} \quad (25)$$

4. EXPERIMENTAL

For this paper, data collected by a Fluxgate magnetic sensor is used. The sensor is fixed by a structure inside a vehicle and the vehicle is rotating with a small radius in a level environment and then the acquired data are collected and stored with a computer. Whereas the vehicle is moving in a level surface, therefore the acquired data are actually forming a plane which the value of its components which are aligned with z axis are almost constant and therefore we can assume them on a plane having a minor angle with the plane xy. Thus, calibration is practically applied one the two x and y axes, thus, the value for the third component cannot be calibrated.

After implementation of the algorithm in Matlab software, our data are divided into two categories. One that is used for the calibration and the other category that is used for examining the results acquired from the previous section and to evaluate the Calibrated Matrices. Only the first 300 points are used for calibration at the first simulation. Figure. 2 represents normalized output of the magnetic vector aligned with x axis and y axis for the abovementioned 300 points.

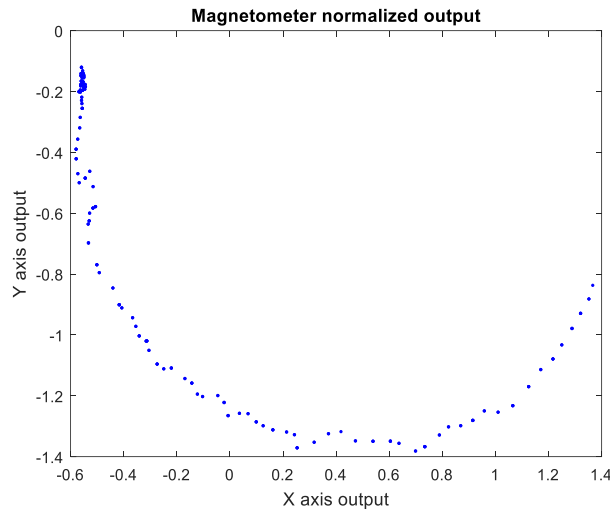


Fig. 2. Magnetometer Normalized output for 300 points.

As it is evident in the Figure 2, the magnetometer is only rotated within 180 degrees for these data. However, if we acquire Calibration Matrices using the data shown in Figure 2, and then to calibrate all of data using those Matrices, the outcome is be as Figure 3:

Whereby the calibration matrix is achieved as:

$$A = \begin{bmatrix} 0.8996 & -0.0403 \\ -0.0403 & 0.8285 \end{bmatrix}$$

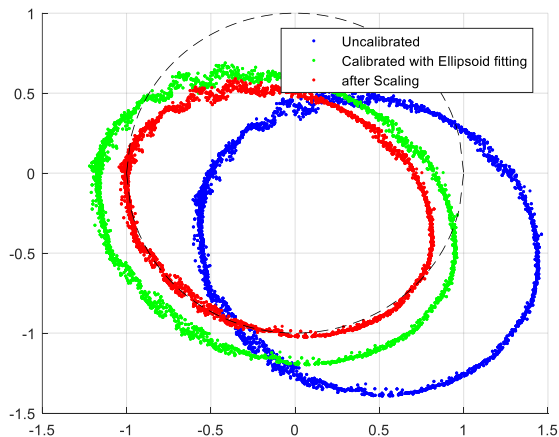


Fig. 3. Calibration Results for 300 points.

In Figure 3, main data is shown in blue. As it can be seen, these data show a diversion of the origin and the geometric position of the points are one an ellipsoid. This Figure clearly displays the importance of calibration. The points in green, represent calibrated data using the acquired Matrices from the 300 points displayed in Figure 3. In order to reconcile all of the outcomes resulted from this method on a unit circle, we need to multiply them by a constant coefficient. We will use inverse value of one of the calibrated points in order to calculate the coefficient and eventually such coefficient will be multiplied by the Calibration Matrix A . Point in red, represent the final calibrated data. As we can see in Figure 3, the output is exactly reconciled on a unit circle only where the initial data used for calibration were present and for the other points, the calibration is not working properly.

Further, we use 600 points for calibration instead of 300 points.

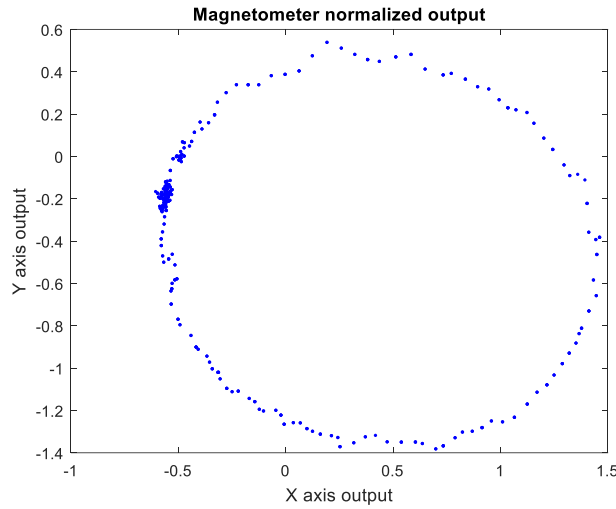


Fig. 4. Magnetometer Normalized output for 600 points

As it is shown in Figure 4, the points that are used for calibration are covering the whole surface of the circle. The result of calibration for the whole dataset using the above data is shown in Figure 5.

Whereby the calibration matrix is achieved as:

$$A = \begin{bmatrix} 0.9968 & 0.0433 \\ 0.0433 & 1.0775 \end{bmatrix}$$

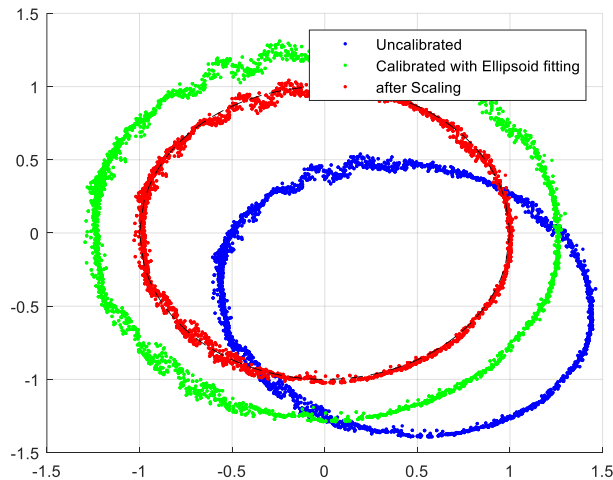


Fig. 5. Calibration Results for 600 points.

It is observed that as the points used for calibration increases that cover the entire surface of the initial ellipsoid, the outcome of calibration are completely reconciled on a unit circle.

At the next stage, we increase the number of data used for calibration to 1800.

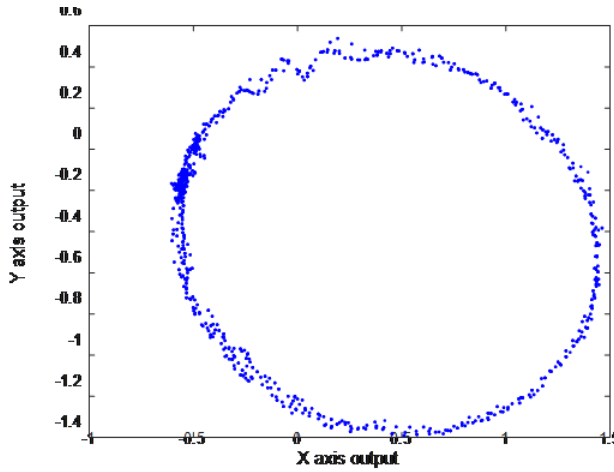


Fig. 6. Magnetometer Normalized output for 1800 points.

In this case, the outcome of calibration will be as displayed in Figure 7:

Whereby the calibration matrix is achieved as:

$$A = \begin{bmatrix} 0.9953 & 0.0502 \\ 0.0502 & 1.0838 \end{bmatrix}$$

$$A = \begin{bmatrix} 0.9953 & 0.0502 \\ 0.0502 & 1.0838 \end{bmatrix}$$

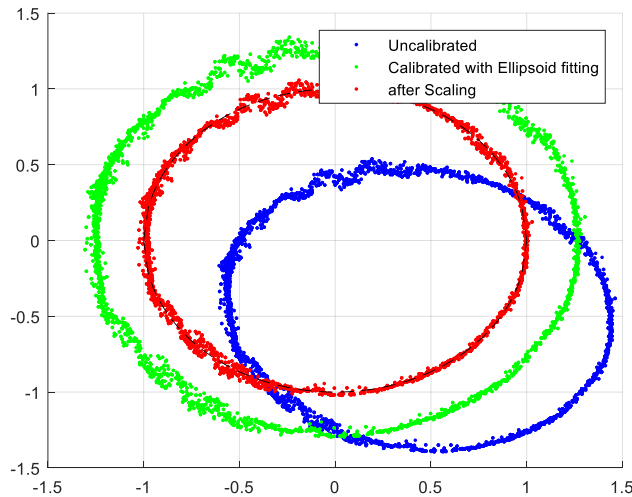


Fig. 7. Calibration Results for 1800 points.

Considering the Figure 7 and while comparing it to the figure for calibrated ones at the previous stage (i.e. with 600 points), we can conclude that there is no significant difference for most of the points between the two states. Only in the fourth quarter of the coordinate plan we can observe a significant optimization. Figure 8 displays a comparison between these two sections:

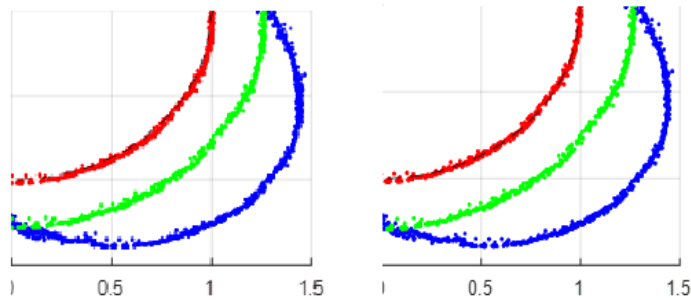


Fig. 8. (a) Calibrated data with 600 calibration points (b) calibrated data using 1800 calibration points

On the right, calibrated data using 1800 calibration points are displayed and on the left calibrated data with 600 calibration points is displayed. As it is evident in the figure, the data displayed on the right are totally reconciled on a unit circle (black dashed line) while the data on the left have a minor difference with the unit circle

5. CONCLUSION

Applying numerical methods for calibration of a magnetometer can largely compensate for the error of deviation from the origin and the error of non-proportionality caused by different sensitivities of the sensors and or presence of external factor. Calibration Matrices can be formed even using only a limited number of measurement data that are collected at the primary phase and then to calibrate the rest of data using such Matrices. As it is displayed in the simulations, data for the calibration stage have a major importance for correctness of the Calibration Matrices. Also, it can be concluded that we can reach the desired output using the minimum number of calibration data that are covering all directions and therefore additional data do not significantly influence the optimization of the performance.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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