ISSN Online: 2821-1693

Transactions on Machine Intelligence

Journal Homepage: https://tmachineintelligence.ir/index.php/TMI

Formation Control of Wheeled Mobile Robots in Chained Form Using Potential Functions

F. Kheradmand Motaghi*,[1]

[1]Department of Mathematics, Iran University of Science and Technology, Tehran, Iran

ARTICLE INFO

Article History:
Received 29 November 2021
Received in revised form
15 January 2022
Accepted 3 March 2022
Available online 5 March 2022

Keywords:
Formation control, multi agents, potential functions, wheeled mobile robots, non-holonomic chained systems.

ABSTRACT

In recent years, using autonomous mobile robots in a multi agent and intelligent environment, designing the formation control is of particular importance. In this paper, after a preliminary introduction of chained systems and potential functions that have been applied in the design of the formation control. Using the goal and collision avoidance potential functions, the formation control of a group of systems whose equations of motion are in the form of chained systems is designed. Considering the conditions of the problem, the multiple agents whose dynamical systems are in the equation of a chained systems, has a limited control design in such a way that different agents are at the start of movement in different status and do not collide along the path of motion and ultimately reach the intended destination by using goal and collision avoidance potential functions. In the following, with the help of the proposed model, the control path is designed for a group of wheeled robots and its control and linear velocity and angular velocities are obtained. Then, simulation of eight wheeled mobile robots with chained dynamics, each of them at the initial are on a unit circle environment, is carried out to the desired goals where the octahedron is a regular octagon. The results obtained from the simulation that the robots reach their desired goals without collision with the other robots indicate the accuracy and efficiency of the suggested method in the design of the formation control for a group of robots in chained form, including wheeled mobile robots.

1. INTRODUCTION

Using autonomous systems in a multi-agents and intelligent environment, the design of their formation control is of particular importance. Control of systems from the very beginning was an essential and vital task for humanity, and many of the phenomena were controlled by the unknown agents. With the industrial revolution in the world in the early twentieth century and the rapid growth of mechanical systems, then electrical systems, and today, mechatronic systems have been critical to controlling these systems. The formation control of mobile robots has become very interesting in recent years. Because of this that control design for a group of robots can have many applications in real life that can change the future transport system. With the automation of the transportation system

* Corresponding Author: Kheradmand.1987@gmail.com
Department of Mathematics, Iran University of Science and Technology, Tehran, Iran

http://dx.doi.org/10.47176/TMI.2022.1

and in the rest of the industry, which in most cases requires the cooperation of a group of robots with specific features, the effective design of the multi-robot control will be very valuable. The main concentration of this work is on the formation control of mobile robots.

In this regard, there are various methods and virtual structure based on the design of the control of the multi-robots, some of them which are most popular for researching [1], [2]. At the moment, international researchers are mostly interested in ground autonomous mobile robots [3], [4], [5], [6], underwater autonomous robots, satellites, and unmanned aerial vehicles [7]. The virtual structure approach [8], [9], leader follower method [11], behavior-based method [21], graph theory method, and artificial potential field formation method are the basic formation control algorithms. Each has its own set of advantages and disadvantages. However, one novel technique employs behavioral approaches, in which the control path for multi-robots can be constructed by creating the goal potential and collision avoidance potential functions.

A good application of formation control to gradient climbing using the potential field approach [16] and Lyapunov's direct method [17] is recently documented in [18]. Nonetheless, the eventual formation layout cannot be predicted. Decentralized systems, on the other hand; see, for example, [19] and [20], require less computational effort and are more scalable to team size. [10, 12] propose a novel universal potential function for designing restricted cooperative controllers for formation stabilization of a collection of mobile agents with confined sensing ranges. In [13], [14], [15], it can also be considered in the design of the dynamic position of the system in the presence of position and speed and acceleration and used in a single simple dynamics of robots, which definitely have robots in the industry and dynamic nature of the complex. And in most control design designs, robots are considered to be free of obstacles, and the obstacles themselves can be either fixed or movable or a combination of both.

In this work, the purpose is to design a formation control that is more compatible with the real world. In this way, a chained system is considered as the dynamics of the system and its corresponding potential functions are designed and then by using the model presented in [10], [12]. The control inputs of a chained system is generally designed and the simulations performed in MATLAB also confirm the pattern's accuracy, and since the actual dynamics of the real robots can be set to a non-holonomic chained systems. Therefore, the non-holonomic wheeled control model is designed with the proposed model and its control inputs, including the linear velocity of the rear wheel and the angular velocity of the front wheel, are obtained and the corresponding simulations are also carried out in MATLAB. Simulation done shows that each of the eight wheeled robots whose initial points are on a circle without interfering with each other towards the specified target, and also the control inputs have relative stability, which show the accurate design of the control inputs.

In the second section, the preconditions, which include the chained systems, potential functions are presented. In the third section, the inputs of the control of the chained system are obtained using potential functions. In the fourth section, the proposed model for a wheeled robots with a non-holonomic chained dynamics is used and its control inputs are obtained. In the final section, the simulations for eight wheeled mobile robots with chained dynamics and non-holonomic constraints of the two inputs has been done, and finally a brief summary is presented.

2. PRELIMINARIES

2.1. Chained Systems

Since most Non-holonomic mechanical systems are either in chained form or have the ability to convert to a chained form, such systems are important. A first-order chained system with two inputs in the canonical form is expressed in terms of the differential equations as follow.

$$
\begin{aligned}
\dot{x}_1 &= u_1 \\
\dot{x}_2 &= u_2 \\
\dot{x}_3 &= x_2 u_1 \\
&\vdots \\
\dot{x}_n &= x_{n-1} u_1
\end{aligned}
\tag{1}
$$

Where $(x_1, x_2, x_3, \cdots, x_n)$ is the system state and (u_1, u_2) are the control inputs of the system.

2.2. Potential Function

In this sector, we present the potential functions that are used in the design of the control inputs of the chained system. In general, there are two potential function, goal and collision avoidance potential functions.

A goal potential function that is designed so that causes the agent reaches to its desired target and it will be zero any time agent comes to its ultimate location. An ordinary definition of it for agent k is as follow.

$$\gamma_k = 0.5\left\| q_k - q_{kf} \right\|^2 \tag{2}$$

The collision avoidance potential function have been designed to do not allow agents to collide with each other, and whenever this happens, the function is infinite, and when the agent k reaches the target, its value will be the least possible. The function for agent k is defined as

$$\beta_k = \sum\nolimits_{j \in N_k} \beta_{kj} \tag{3}$$

Where β_{kj} is a function in the form of $\left\| q_{kj} \right\|^2 / 2$ and $\left\| q_{kjf} \right\|^2 / 2$, q_{kf} is the final position of agent k, q_k is the state of agent k, N_k is the set of all agents except agent k and the following relations satisfies.

$$q_{kj} = q_k - q_j, \qquad q_{kjf} = q_{kf} - q_{jf}, \tag{4}$$

Given the existence of many functions that satisfies in the properties of the function β_{kj}, here the following function is considered based on the reference [10].

$$\beta_{kj} = \left(\frac{\left\| q_{kj} \right\|^2 / 2}{\left\| q_{kjf} \right\|^2 / 2} + \frac{1}{\left\| q_{kj} \right\|^2 / 2} - \frac{2}{\left\| q_{kjf} \right\|^2 / 2} \right)^2 \times h_{kj}\left(\left\| q_{kj} \right\|^2 / 2, \; a_{kj}{}^2 / 2, \; b_{kj}{}^2 / 2 \right) \tag{5}$$

And also the function $h(x,a,b)$ is a p (with $p \ge 2$) times differentiable bump function that is defined as following

$$h(x,a,b) = 1 - \frac{\int_a^x f(v-a) f(b-v) dv}{\int_a^b f(v-a) f(b-v) dv} \tag{6}$$

In the above relation, the function $f(y)$ is considered as follows, where p is a positive integer.

$$\begin{cases} f(y) = 0, & if \;\; y \le 0, \\ f(y) = y^p, & if \;\; y > 0 \end{cases} \tag{7}$$

Note: For further investigation about properties of the β_{kj} function, see reference [10] in which the contents of this section has been brought of that.

It is time to define the potential function as follows, in which γ_k and β_k are the goal and collision avoidance potential function of agent k in a group with N agents.

$$\varphi = \sum_{k=1}^{N} (\gamma_k + 0.5\beta_k) \tag{8}$$

2.3. Problem layout

Let's take a group of N mobile agents, all of them are a two-input chained system, whose dynamical equations are as follows.

$$\begin{aligned} \dot{x}_{1k} &= u_{1k} \\ \dot{x}_{2k} &= u_{2k} \\ \dot{x}_{3k} &= x_{2k}u_{1k} \\ &\vdots \\ \dot{x}_{ik} &= x_{(i-1)k}u_{1k} \qquad k = 1,2,3,\ldots,N \qquad i = 4,5,\ldots,n \end{aligned} \tag{9}$$

Where $x_{ik} \in \mathbb{R}$ and $u_{ik} \in \mathbb{R}$ are the status and control inputs of agent k. It is also assumed that $n > 1$ and $N > 1$ as well as every agent is considered as an independent point.

3. CONTROL DESIGN

In the design of this problem, it is assumed a group of N agents, each with the dynamics of the chained system, at an initial time ($t_0 \geq 0$) are in a different position relative to each other, and each of these agents has different target positions. And this, in mathematical terms, means that there are positive constants of ε_1 and ε_2 in which the following relation is established.

$$\begin{aligned} &\|q_m(t_0) - q_n(t_0)\| \geq \varepsilon_1, \quad \|q_{mf}(t_0) - q_{nf}(t_0)\| \geq \varepsilon_2, \\ &\forall (m,n) \in \{1,2,\ldots,N\},\, m \neq n \end{aligned} \tag{10}$$

That $q_{mf}, m = 1,2,\ldots,N$ is the ultimate position of the agent m, In addition, the agent m has ability to gauge its position, as well as find the distance from the rest of the agents in the sphere centered on the $m-$agent and the radius R_m. The goal of theem, so that each agent moves and reaches target -Controller input design is to control the k u_k without interfering with other agents. We have following mathematical expressions:

In addition, the agent m merely has ability to gauge its own status and if the other member of the team are in a sphere, so the agent m can find these members, the sphere is centered at the agent and has a radius of R_m larger than a strictly positive constant. The goal is to design the bounded control input u_m for each agent m so that each agent asymptotically come close to its demanded place while do not collide with the other in the group, i.e. for all agents

$$\begin{aligned} &\|u_m(t)\| \leq \varepsilon_3,\ \lim_{t\to\infty} q_m(t) - q_{mf}(t) = 0, \\ &\|q_m(t) - q_n(t)\| \geq \varepsilon_4, \\ &\forall (m,n) \in \{1,2,\ldots,N\},\, m \neq n \quad \mathrm{t} \geq \mathrm{t}_0 \geq 0 \end{aligned} \tag{11}$$

In which R_m is positive constant and $(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)$ are strictly positive constants now we consider potential function defined in section II as follows

$$\varphi = \sum_{k=1}^{N} (\gamma_k + 0.5\beta_k) \tag{12}$$

By δεριϖατιον of the above function relative to t, we will have

$$\dot{\varphi}_{2k} = \sum_{k=1}^{N} (\Omega_{2k}^T u_{2k}) \tag{13}$$

In which

$$\Omega_{1k} = (q_{1k} - q_{1kf}) + \sum\nolimits_{j\in N_k} \beta'_{1kj} q_{1kj}, \tag{14}$$

We must note that since we are working in a $n-$ dimensional environment, we have

$$\Omega_{1k}=\begin{bmatrix}\Omega_{11}\\ \Omega_{12}\\ \vdots\\ \Omega_{1n}\end{bmatrix}\quad q=\begin{bmatrix}1\\ q_{12}\\ \vdots\\ q_{1n}\end{bmatrix} \tag{15}$$

And therefore we write the following relation

$$\Omega_1^T\dot{q}_1=[\Omega_{11}\ \Omega_{12}\ \ldots\ \Omega_{1n}]\begin{bmatrix}\dot{q}_{11}\\ \dot{q}_{12}\\ \vdots\\ \dot{q}_{1n}\end{bmatrix}= \tag{16}$$

$$\dot{q}_{11}\Omega_{11}+\dot{q}_{12}\Omega_{12}+\ldots+\dot{q}_{1n}\Omega_{1n}$$

With the knowledge of the dynamics of the chained systems and its placement in the relationship (9) we have.

$$\begin{aligned}&\Omega_{11}u_{11}+\Omega_{12}u_{12}+\Omega_{13}q_{12}u_{11}+\Omega_{14}q_{13}u_{11}+\ldots+\Omega_{1n}q_{1n-1}u_{11}\\ &=u_{11}(\Omega_{11}+\Omega_{13}q_{12}+\Omega_{14}q_{13}+\ldots+\Omega_{1n}q_{1n-1})+u_{12}(\Omega_{12})\end{aligned} \tag{17}$$

In order to construct the control inputs, the function ψ which is a scalar differentiable and bounded function and satisfies below constraints has been considered.

$$\begin{aligned}&1)\ \ |\psi(x)|\le M_1\\ &2)\ \ \psi(x)=0,\quad if\quad x=0,\qquad x\psi(x)\succ 0,\qquad if\quad x\ne 0\\ &3)\ \ \psi(-x)=-\psi(x),\qquad (x-y)(\psi(x)-\psi(y))\ge 0,\\ &4)\ \ \left|\frac{\psi(x)}{x}\right|\le M_2,\qquad \left|\frac{\partial\psi(x)}{\partial x}\right|\le M_3,\qquad \frac{\partial\psi(x)}{\partial x}\|_{x=0}=1\end{aligned} \tag{18}$$

In the above equations $x,y\in\mathbb{R},\ M_1,M_2\ and\ M_3$ are strictly positive constant and also

$$\Psi(\Omega_{1k})=[\psi(\Omega_{1k}^1)\ \psi(\Omega_{1k}^2)\ \psi(\Omega_{1k}^3)\ \ldots\ \psi(\Omega_{1k}^1)\ \ldots\ \psi(\Omega_{1k}^1)]^T \tag{19}$$

The function Ψ can be of type $\arctan x$ and $\tanh x$. For the first agent of the N agents, through the potential function and the resulting relations, the control inputs is designed as follows.

$$u_{11}=-c\Psi[\Omega_{11}+\Omega_{13}\mathrm{q}_{12}+\Omega_{14}\mathrm{q}_{13}+\ \ldots\ +\Omega_{1n}q_{1(n-1)}]$$

$$=-c[\varphi(\Omega_{11})\varphi(\Omega_{13})\varphi(\Omega_{14})\ldots\varphi(\Omega_{1n})]\begin{bmatrix}1\\ q_{12}\\ q_{13}\\ \vdots\\ q_{1n-1}\end{bmatrix} \tag{20}$$

$$u_{12}=-c\Psi(\Omega_{12})=-c\varphi(\Omega_{12})$$

Note: In the control input design, the following equations have been used.

$$\Omega_i = [\Omega_{i1}\ \Omega_{i2}\ \dots\ \Omega_{in}] = (q_i - q_{if}) + \sum_{j\in N_i} \beta'_{ij} q_{ij} =$$
$$[q_{i1}\ q_{i2}\ q_{i3}\ \dots\ q_{in}] - [q_{i1f}\ q_{i2f}\ q_{i3f}\ \dots\ q_{\inf}] +$$
$$\sum_{j\in N_i} \beta'_{ij}[q_{ij1}\ q_{ij2}\ q_{ij3}\ \dots\ q_{ijn}] =$$
$$[(q_{i1} - q_{if1}) + \sum_{j\in N_i} \beta'_{ij} q_{ij1}\ \ (q_{i2} - q_{if2}) + \sum_{j\in N_i} \beta'_{ij} q_{ij2}\ \dots$$
$$(q_{in} - q_{ifn}) + \sum_{j\in N_i} \beta'_{ij} q_{ijn},]$$
(21)

In other words, we have that

$$\begin{aligned} \Omega_{i1} &= (q_{i1} - q_{if1}) + \sum_{j\in N_i} \beta'_{ij} q_{ij1}, \\ \Omega_{i2} &= (q_{i2} - q_{if2}) + \sum_{j\in N_i} \beta'_{ij} q_{ij2}, \\ &\vdots \\ \Omega_{in} &= (q_{in} - q_{ifn}) + \sum_{j\in N_i} \beta'_{ij} q_{ijn}, \end{aligned} \tag{22}$$

4. APPLICATION TO CAR-LIKE MOBILE ROBOTS

The dynamic model of a car-like robot as shown in Fig. 1. Powered by a front-wheel drive and two-wheeled rear-wheel drive, is as follows:

$$\begin{cases} \dot{x} = V\cos\theta \\ \dot{y} = V\sin\theta \\ \dot{\theta} = \dfrac{V}{d}\tan\varphi \\ \dot{\varphi} = \omega \end{cases} \tag{23}$$

Where we have followings:

$(x, y) \rightarrow$ The posture of the robot, which are located in the center of the rear wheel.

$\theta \rightarrow$ Rotation angle of the robot relative to the X-axis

$\varphi \rightarrow$ Steering angle

$V \rightarrow$ The linear velocity of the robot;

$\omega \rightarrow$ Steering rate of the robot wheel

$d \rightarrow$ The wheel-base

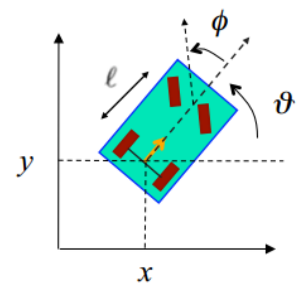

Fig. 1. Formation coordinates in 2-D.

The system is known as a Non-Holonomic chained system, the condition of the system as well as the coordinates of the system are as follows. The condition is that the robot does not drown during the move.

$$q^T = [x\ y\ \theta\ \varphi]$$

$$A(q)\dot{q} = \begin{pmatrix} -\sin\theta & \cos\theta & 0 & 0 \\ -\sin(\theta+\varphi) & \cos(\theta+\varphi) & d\cos\theta & 0 \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi} \end{pmatrix} = 0, \tag{24}$$

And also, the form of the chained system is as follow. Using the local coordinates transformation

$$x_1 = x,\ \ x_2 = \frac{\tan\varphi}{l\cos^3\theta},\ \ x_3 = \tan\theta,\ \ x_4 = y, \tag{25}$$

And by replacing corresponding input, system (15) can be written in the extended chained form (3)(4) for $n = 4$

4.1. Designing the control inputs for Car-Like Robot

In this sector, we intend to use the proposed model in the section three To design the control inputs of a group of car-like robots that was introduced in the previous section, Since we know that the wheeled mobile robot has a chained dynamic system with two inputs for the linear velocity of the rear wheel and the pivot speed of the front robotic wheel. To do this, we define the following relationships that include robot status and potential functions.

$$\begin{aligned} &q_i = [x_i\ y_i\ \theta_i\ \varphi_i], \quad \beta_i = \sum\nolimits_{j\in N_i} \beta_{ij}, \\ &\gamma_i = 0.5\left\|q_i - q_{if}\right\|^2, \quad \varphi = \sum_{k=1}^{N} (\gamma_k + 0.5\beta_k) \end{aligned} \tag{26}$$

As with the Section three, we derive from the potential function and using the previous relations we obtain

$$\begin{aligned} &\dot{\varphi} = \sum_{i=1}^{N} (\Omega_i^T \dot{q}_i) = [\Omega_1\ \Omega_2\ \Omega_3\ \Omega_4] \begin{bmatrix} \dot{q}_{i1} \\ \dot{q}_{i2} \\ \dot{q}_{i3} \\ \dot{q}_{i4} \end{bmatrix} = \\ &\Omega_1^T \dot{q}_{i4} + \Omega_2^T \dot{q}_{i2} + \Omega_3^T \dot{q}_{i3} + \Omega_4^T \dot{q}_{i4} \end{aligned} \tag{27}$$

By placing the equations (23) In the above equations and the factoring, the two control inputs is presented as follows. (It should be noted that all the functions introduced in previous sections including the functions β_{ij} ψ ، are also used here).

$$\begin{aligned} &\Omega_1 V\cos\theta + \Omega_2 V\sin\theta + \Omega_3 V / d\tan\varphi + \omega(\Omega_4) = \\ &V(\Omega_1\cos\theta + \Omega_2\sin\theta + \Omega_3 1/d\tan\varphi) + \omega(\Omega_4) \\ &V = -c\Psi(\Omega_1\cos\theta + \Omega_2\sin\theta + \Omega_3 1/d\tan\varphi) \\ &\omega = -c\Psi(\Omega_4) \end{aligned} \tag{28}$$

In the above relations, the below relationships have been used in designing control inputs That's for clarification they have been brought here

$$\begin{aligned}&\Omega_i=[\Omega_1\ \Omega_2\ \Omega_3\ \Omega_4]=(q_i-q_{if})+\sum_{j\in N_i}\beta'_{ij}q_{ij}=\\&[x_i\ \mathrm{y}_i\ \theta_i\ \varphi_i]-[x_{if}\ \mathrm{y}_{if}\ \theta_{if}\ \varphi_{if}]+\sum_{j\in N_i}\beta'_{ij}[x_i\ \mathrm{y}_i\ \theta_i\ \varphi_i]\\&=[(x_i-x_{if})+\sum_{j\in N_i}\beta'_{ij}x_{ij}\quad (y_i-y_{if})+\sum_{j\in N_i}\beta'_{ij}y_{ij}\\&(\theta_i-\theta_{if})+\sum_{j\in N_i}\beta'_{ij}\theta_{ij}\quad(\varphi_i-\varphi_{if})+\sum_{j\in N_i}\beta'_{ij}\varphi_{ij}]\end{aligned}\tag{29}$$

And the following functions are resolved which have been used in relation (28)

$$\begin{aligned}&\Omega_{i1}=(x_{i1}-x_{if1})+\sum_{j\in N_i}\beta'_{ij}x_{ij},\\&\Omega_{i2}=(y_{i1}-y_{if1})+\sum_{j\in N_i}\beta'_{ij}y_{ij}\\&\Omega_{i3}=(\theta_{i1}-\theta_{if1})+\sum_{j\in N_i}\beta'_{ij}\theta_{ij}\\&\Omega_{i4}=(\varphi_{i1}-\varphi_{if1})+\sum_{j\in N_i}\beta'_{ij}\varphi_{ij}\end{aligned}\tag{30}$$

5. SIMULATIONS

In this section, we have done simulations with eight wheeled mobile robots in a two-dimensional environment $(n=2,\ N=8)$. The Robots are set up randomly in a unit circle, which it's center in origin with radius one. The desired formation of robots is specified in shape, location and orientation as $q_{if}=(x_{if},y_{if},\theta_{if},\varphi_{if})=R_f(\sin(i-1)2\pi/N,\cos(i-1)2\pi/N,(i-1)2\pi/N,0)$, $i=1,2,\ldots,N$ with $R_f=10,$ i.e., the demanded formation for robots is a polygon and it's vertices is located on a circle with center of origin and radius of ten. The sensing range of all robots is $R_i=2,$. The function β_{ij} is selected as in (5). The p times differential bump functions have $p=2,\ a_{ij}=1,\ b_{ij}=1.50,$. The function $\psi(x)$ and control gain consecutively are chosen as $\tanh(x)$ and $c=2$. Simulation results show that wheeled mobile robots move slowly toward the targets without colliding with each other. Figure 2. Shows the direction of movement of eight robots from initial points to targets. Figure 3. Shows the distance of each robot from the start of the trajectory to reaching the target, which, over time, carries out to zero, which means that the robots move towards the target, and Figure 4. Shows the distance between robots from start to the end of the trajectory shows that all distances are larger than zero, which means that the robots do not collide while they are moving to the desired goals.

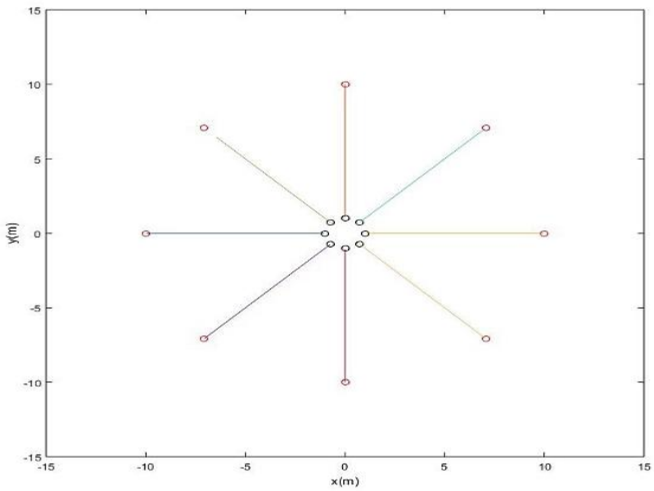

Fig. 2. Trajectory of eight wheeled mobile robots

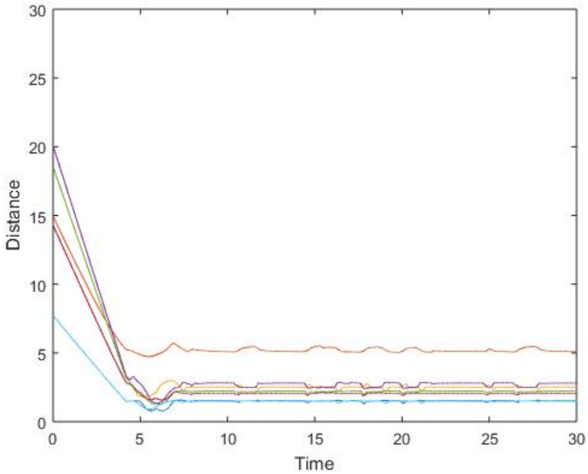

Fig. 3. Distances among robots duration simulation

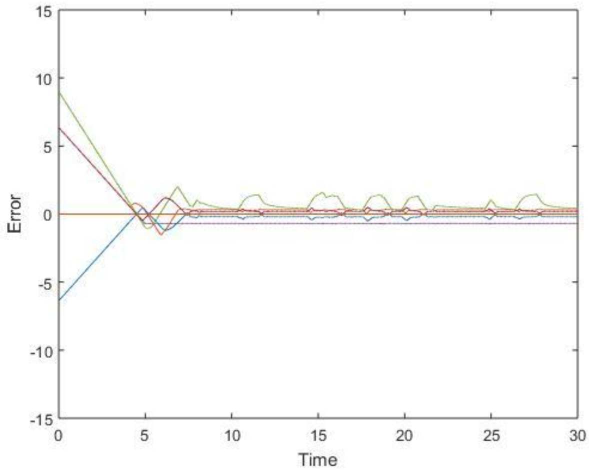

Fig. 4. Distances of robots respect to desired destination

6. CONCLUSION

According to the model presented in this paper for the design of the multi-agent formation control, in this research, using the goal and collision avoidance potential functions, the formation control for these agents was designed. Also, the proposed model for non-holonomic wheeled mobile robots has been used and it's linear and angular velocity as control inputs are obtained and simulation for eight wheeled mobile robots has been done to show accuracy of presented model. The obtained model for multi-agent systems can be studied and used in the presence of fixed and moving obstacles, as well as the stability of such systems with this formation control model as a theoretical analysis is the future interest works of the author, which is likely to be a more stable and precise algorithm.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

REFERENCES

[1] Ganguli, A., Cortés, J., & Bullo, F. (2009). Multirobot rendezvous with visibility sensors in nonconvex environments. IEEE Transactions on Robotics, 25(2), 340–352. https://doi.org/10.1109/TRO.2009.2013493

[2] Wang, X. K., Li, X., & Zheng, Z. Q. (2013). Survey of developments on multi-agent formation control related problems. Control and Decision, 28(11), 1601–1613.

[3] Wang, B. F., Zhang, R. L., Li, S., & Chen, Q. W. (2015). Formation control for car-like mobile robots based on trajectory tracking. Control and Decision, 30(1), 176–180.

[4] Wang, X., Yu, C., & Lin, Z. (2012). A dual quaternion solution to attitude and position control for rigid-body coordination. IEEE Transactions on Robotics, 28(5), 1162–1170. https://doi.org/10.1109/TRO.2012.2196310

[5] Zhang, R., Li, S., & Chen, Q. (2013). Dynamic formation control for car-like mobile robots. Robot, 35(6), 651–656. https://doi.org/10.3724/SP.J.1218.2013.00651

[6] Weili, L., & S. M. G. H. (2005). Tracking control of car-like mobile robot based on Backstopping. Journal of Southeast University (Natural Science Edition), 2.

[7] Xiao, S., Feng, L., Lian, H., & Du, B. (2016, June). Dynamic formation and obstacle avoidance control for multi robot system. In 2016 12th World Congress on Intelligent Control and Automation (WCICA) (pp. 59–63). IEEE. https://doi.org/10.1109/WCICA.2016.7578642

[8] Ghommam, J., Mehrjerdi, H., Saad, M., & Mnif, F. (2010). Formation path following control of unicycle-type mobile robots. Robotics and Autonomous Systems, 58(5), 727–736. https://doi.org/10.1016/j.robot.2009.10.007

[9] Tian, Y. P., & Zhang, Y. (2012). High-order consensus of heterogeneous multi-agent systems with unknown communication delays. Automatica, 48(6), 1205–1212. https://doi.org/10.1016/j.automatica.2012.03.017

[10] Do, K. D. (2007). Bounded controllers for formation stabilization of mobile agents with limited sensing ranges. IEEE Transactions on Automatic Control, 52(3), 569–576. https://doi.org/10.1109/TAC.2007.892382

[11] Yang, F., Liu, S., & Dong, D. (2012). Robot behavior and service-based motion behavior structure design in formation control. Robot, 34(1), 120–128. https://doi.org/10.3724/SPJ.1218.2012.00120

[12] Do, K. D. (2006, June). Formation stabilization and tracking control of mobile agents using local potential functions. In 2006 American Control Conference (pp. 6–pp). IEEE. https://doi.org/10.1109/ACC.2006.1656536

[13] Luo, J., & Tsiotras, P. (2000). Control design for chained-form systems with bounded inputs. Systems & Control Letters, 39(2), 123–131. https://doi.org/10.1016/S0167-6911(99)00097-3

[14] Yin, L., Yin, Y., & Lin, C. J. (2009). A new potential field method for mobile robot path planning in the dynamic environments. Asian Journal of Control, 11(2), 214–225. https://doi.org/10.1002/asjc.98

[15] Chunyu, J., Qu, Z., Pollak, E., & Falash, M. (2009, June). A new reactive target-tracking control with obstacle avoidance in a dynamic environment. In 2009 American Control Conference (pp. 3872–3877). IEEE. https://doi.org/10.1109/ACC.2009.5160362

[16] Leonard, N. E., & Fiorelli, E. (2001). Virtual leaders, artificial potentials and coordinated control of groups. In Proceedings of the 40th IEEE Conference on Decision and Control (Vol. 3, pp. 2968–2973). IEEE. https://doi.org/10.1109/CDC.2001.980728

[17] Ogren, P., Egerstedt, M., & Hu, X. (2002). A control Lyapunov function approach to multiagent coordination. IEEE Transactions on Robotics and Automation, 18(5), 847–851. https://doi.org/10.1109/TRA.2002.804500

[18] Ogren, P., Fiorelli, E., & Leonard, N. E. (2004). Cooperative control of mobile sensor networks: Adaptive gradient climbing in a distributed environment. IEEE Transactions on Automatic Control, 49(8), 1292–1302. https://doi.org/10.1109/TAC.2004.832203

[19] Stipanović, D. M., Inalhan, G., Teo, R., & Tomlin, C. J. (2004). Decentralized overlapping control of a formation of unmanned aerial vehicles. Automatica, 40(8), 1285–1296. https://doi.org/10.1016/j.automatica.2004.02.017

[20] Tanner, H. G., & Kumar, A. (2005, June). Formation stabilization of multiple agents using decentralized navigation functions. In Robotics: Science and Systems (Vol. 1, pp. 49–56).

[21] Ren, L., Wang, W. D., & Du, Z. J. (2013). Dynamic and optimized formation switching for multiple mobile robots in obstacle environments. Robot, 35(5), 535–543. https://doi.org/10.3724/SP.J.1218.2013.00535