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A Niching Ring Topology Genetic Algorithm for Multimodal Optimization

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ABSTRACT

Multimodal optimization represents a significant and ongoing challenge within the broader field of optimization, particularly due to the presence of multiple global and local optima within a complex search space. Unlike unimodal problems that focus on a single optimal solution, multimodal problems require algorithms to locate and maintain a diverse set of high-quality solutions across various regions of the landscape. This characteristic reflects many real-world scenarios, such as engineering design, robotics, and bioinformatics, where multiple viable solutions can coexist. Traditional optimization algorithms often struggle in such settings, as they tend to converge prematurely to a single optimum and lack mechanisms for diversity preservation. In this paper, we propose a novel niching-based Genetic Algorithm (GA) tailored specifically for multimodal optimization problems. The proposed algorithm dynamically forms niches based on the spatial distribution of individuals in the population, enabling the preservation and evolution of multiple optima simultaneously. To ensure that niches are maintained effectively, the genetic operators are strategically modified to minimize disruption to niche structure during crossover and mutation. Extensive experiments conducted on standard multimodal benchmark functions demonstrate that our approach consistently outperforms existing methods in both convergence speed and solution diversity. The results validate the algorithm's robustness and its practical potential in solving complex multimodal problems.

1. INTRODUCTION

Multimodal optimization, a formidable challenge in the realm of optimization algorithms, pertains to the identification and exploration of multiple optimal solutions within complex search spaces. Addressing this intricate problem has far-reaching implications across various disciplines, from engineering design to machine learning. This paper provides a comprehensive survey of contemporary multimodal optimization techniques, emphasizing recent developments and strategies employed to enhance their efficacy [1-4].

Niching methods discover multiple optima in a single iteration by dividing the main population into subpopulations, or neighborhoods. Each neighborhood aims to explore a specific part of the search space in order

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to preserve the necessary diversity within the main population. Numerous studies have been conducted on niching methods within the framework of evolutionary algorithms [5].

Crowding is among the first methods deemed effective in addressing multimodal problems. De Jong presented the initial draft of the crowding technique in his publication [6]. This method involves cross-checking an offspring with various individuals randomly selected from the present population, with the objective of replacing the offspring with the most corresponding candidate. The number of these candidates must be predetermined by the user and is referred to as crowding factor. Crowding's primary benefit is its simplicity, while its principal disadvantage is replacement error [7]. Original crowding finds limited success in solving multimodal problems. To enhance the crowding technique, deterministic crowding has been introduced, which eliminates the crowding factor parameter [8]. Compared to the original crowding approach, deterministic crowding lowers the replacement error and increases selection pressure [7]. Wong and colleagues (2010) integrated crowding and the principle of locality into the Differential Evolution framework to create new offsprings using spatial and temporal locality [9]. Another niching method that behaves similarly to crowding is Restricted Tournament Selection (RTS), where the offspring is compared to a set of w random samples based on their Euclidean or Hamming distance. The closest individual to the offspring is chosen to compete for survival in the subsequent generation. RTS maintains significant diversity during the search process. However, the size of the window (w) and its replacement error, as well as the need for predefinition, are RTS's primary limitations [10].

Fitness sharing is another type of niching which can be considered among the most popular ones [11]. In this method, individuals share their search information with other individuals which drop in the same neighborhood. Such a strategy gives enough strength to the algorithm to form and preserve the stable subpopulations. However, determining the size of neighborhood to share the information, noted by σ share, can be difficult and problem-dependent. Fitness Euclidean-distance Ratio (FER) can be considered as a type of fitness sharing method as employed in FERPSO [12]. In FERPSO global best in velocity update formula of PSO is replaced with a type of particle's local best, i.e. the particle with the largest FER value among neighbor particles. FER values are calculated as follows:

$$FER_{(j,i)} = \alpha \frac{f(p_j) - f(p_i)}{\left\| p_i - p_i \right\|} . \tag{1}$$

where $f(p_j)$ and $f(p_i)$ are the personal bests of the j_{th} and i_{th} particles, respectively. In scaling factor $= \alpha \frac{\|s\|}{f(\mathrm{p}_g) - f(\mathrm{p}_w)}$, p_g is p_w the best and is the worst particle in the current population. $\|s\|$ represents the size of search space, which can be determined based on the difference between lower and upper bound of problem. This procedure makes particles available to interact with each other. There are other works on fitness sharing, for instance in [13], an FER method, based on DE is introduced as FERDE. In FERDE, the individuals that produce mutant vector are being selected based on their FER value.

There are a number of other niching methods which try to preserve diversity through eliminating the unsuitable individuals and maintaining just the best individual (or a number of better ones) in each niche; these strategies are known as clearing. In clearing, usually the best individual at a niche is selected and then the individuals which have lower distance from the best (based on the σ_{clear} parameter) will be eliminated; in other words, clearing method tries to eliminate similar individuals which cannot give further information about functional landscape, and thereby leads to maintain the best individuals in each niche. Clearing is also simpler than other niching methods such as sharing [7]. However, predefining a parameter σ_{clear} can be as difficult as σ_{share} in fitness sharing method. The order of computational complexity of different niching methods have been compared in [14]; clearing has the most running time in comparison with other niching methods [14]. Furthermore, since similar individuals cannot survive with each other, the convergence speed of clearing can be very slow [7].

Speciation is another popular type of niching methods. In this method, species form niches in the main population. All individuals in the same neighborhood are known as a cluster; the center of the cluster is called the species seed. Individuals with Euclidean distances of lower than *rs* (species radius) from the species seed will form a species. Ability to preserve high diversity and stable niches during the search process, and the dependence on *rs* parameter are the main points of strength and weakness of speciation, respectively. A species conservation strategy in the framework of genetic algorithm has been proposed in [15]. Species-based PSO (SPSO) has been introduced by Li [16, 17]. In each iteration of SPSO, the species seeds are determined from the entire population and form different species groups separately. To form these species, a niche radius must be specified in order to determine the bound of each species. In more recent studies, a speciation parameter-free method which is used the search history to form species has been introduced [18].

Mutation is essentially one of the other important tools in EAs to provide diversity during the search process in multimodal optimization. Wang et al. in [19] proposed an adaptive mutation strategy to maintain the diversity in the main population. In this method, three types of mutation (i.e. Cauchy, Levy, and Gaussian) are employed: the first two are used in the first steps while the last one being employed in the final steps. The algorithms switch between them in such an adaptive way that facilitates a fine tradeoff between exploration and exploitation abilities. Another mutation strategy has been introduced in [7] based on the framework of DE for multimodal optimization. The introduced mutation in [7] performs within each Euclidean neighborhood in which a parameter specifies the size of this neighborhood. Most past studies have utilized niching parameters to construct a niche with values of σclear and σshare. However, [4] has illustrated issues with using niching parameters. To avoid these problems, Li [5] introduced ring topology PSO as an interesting niching method called lbest PSO. In this approach, each particle interacts solely with its immediate neighbors. There are four distinct variations of lbest PSO: r2pso, r3pso, r2psolhc, and r3psolhc. In r2pso, each particle interacts solely with its immediate right neighbor. In r3pso, however, particles interact with both their closest left and right neighbors. Consequently, lbest PSO obviates the need for users to specify niching parameters. Although r2psolhc and r3psolhc are both similar to r2pso and r3pso, the former techniques differ in the way they handle neighborhood overlapping. Distance-based Locally Informed Particle Swarm Optimization (LIPS) is a parameter-free niching method proposed by Qu et al. (20). In LIPS, particles exchange information with their nearest neighbors based on Euclidean distance, and the neighborhood size changes during the search process. Local bests are used in the PSO updating formula instead of global best.

Most current multimodal optimization methods lack fine local search ability. This can hinder the achievement of desired optima even when the cases are already converged in proximity, particularly with high expected accuracy. To enhance local search ability in multimodal optimization, memetic algorithms have been developed. These algorithms combine an EA and some local searches. In [21], PSO and a modified version of Broyden-Fletcher-Goldfarb-Shanno are incorporated. Vitela et al. [22] have created a GA fused with a hill-climbing gradient-based algorithm. The study conducted by Qu et al. (2013) is a significant contribution to multimodal optimization using memetic algorithms. Qu's proposed memetic algorithm (2013) employs a local search over all personal bests of individuals, which is the most commonly-used approach in multimodal optimization utilizing memetic algorithms [20]. Qu's proposed memetic algorithm (2013) employs a local search over all personal bests of individuals, which is the most commonly-used approach in multimodal optimization utilizing memetic algorithms [20]. Utilizing this approach can significantly improve the precision of the final solutions. Unfortunately, the algorithm's time complexity has increased. Additionally, performing a local search across all individuals in the population could lead to premature convergence in subpopulations. To address this issue, our paper introduces a new combination strategy for local search in multimodal algorithms. Our proposed method improves both the solutions' quality and the algorithms' time complexity. The remainder of the paper is organized as follows. The niching Genetic Algorithm (GA) proposed in Section 2 is described. Section 3 presents the experimental results and comparisons, and Section 4 provides concluding remarks.

2. PROPOSED NICHING GA

In this section, we propose a niching GA for multimodal optimization. The population in the present paper was formed by ring topology. The structure of population is represented in Fig. 1(a). Each individual interacts only with immediate neighbors. The type of interaction in the proposed method is defined in terms of crossover. Neighbors of an individual are those with the nearest index. The following are the four variants of the proposed method used in the experiments:

- R3GA: a ring topology GA; each individual interacts with its immediate next and previous neighbors in terms of index position.
- R2GA: a ring topology GA; each individual interacts with its immediate next neighbor in terms of index position.
- R3GA-lhc: the same as r3GA, but with no overlapping neighborhoods.
- R2GA-lhc: the same as r2GA, but with no overlapping neighborhoods.
- The r3GA-lhc and r2GA-lhc variants behave as local hill climbers, and are, therefore, more suitable for finding the global as well as local optima. The neighborhoods in r3GA and r3GA-lhc are shown in Fig. 1(b).
- To prove the power of our proposed method, we used well-known genetic operators, as addressed in the following subsections.

2.1. Crossover

Crossover in GA is the most important operator constructing new generation from the current individuals. In our proposed method, the concept of interaction between individuals was provided by crossover. Eq.1 describes the crossover in the proposed method.

$$offspring_i = I + \left(r1 * Best_i\right) - \left(r2 * X_i\right) \tag{2}$$

where X_i is the ith individual, $Best_i$ is the best individual in the i^{th} neighborhood, and I is a random with normal distribution in $N(X_i, \sigma)$.

2.2. Mutation

A simple mutation operator was employed in the present paper; an individual randomly changes in the predefined range, with the probability of Pm (mutation probability)

2.3. Selection

New population formed by selecting the best individual among new offsprings were generated with genetic operators and the current population according to their fitness values.

$$X_i(t+1) = Best\{X_i(\mathrm{t}), offspring_i\} \tag{3}$$

Table 1. Benchmark functions used in comparisons.

Function	Test function	Peaks
F1: Two-Peak Trap	$f_1(x) = \begin{cases} \frac{160}{15}(15 - x), & 0 \le x \le 15 \\ \frac{200}{5}(x - 15), & 15 \le x \le 20 \end{cases}$	1
F2: Central Two-Peak Trap	$f_2(x) = \begin{cases} \frac{160}{10}, & 0 \le x \le 10 \\ \frac{160}{5}(15 - x) & 10 \le x \le 15 \\ \frac{200}{5}(x - 15) & 15 \le x \le 20 \end{cases}$	1

F3: Five-Uneven-Peak Trap	$f_3(x) = \begin{cases} 80(2.5-x) & 0 \le x < 2.5 \\ 64(x-2.5) & 2.5 \le x < 5 \\ 64(7.5-x) & 5 \le x < 7.5 \\ 28(x-7.5) & 7.5 \le x < 12.5 \\ 28(17.5-x) & 12.5 \le x < 17.5 \\ 32(x-17.5) & 17.5 \le x < 22.5 \\ 32(27.5-x) & 22.5 \le x < 27.5 \\ 80(x-27.5), & 27.5 \le x \le 30 \end{cases}$	2
F4: Equal Maxima	$f_4(x) = sin^6(5\pi x)$	5
F5: Decreasing Maxima	$f_5(x) = exp\left[-2\log(2).(\frac{x-0.1}{0.8})^2\right] sin^6(5\pi x)$	1
F6: Uneven Maxima	$f_6(x) = sin^6(5\pi(x^{3/4}-0.05))$	5
F7: Uneven Decreasing Maxima	$f_7(x) = exp\left[-2\log(2).(\frac{x-0.08}{0.854})^2\right] sin^6(5\pi(x^{3/4}-0.05))$	1
F8: Himmelblau's	$f_8(x,y) = 200-(x^2+y-11)^2-(x+y^2-7)^2$	4
F9: Six-Hump Camel Back	$f_9(x,y) = -4\left[\left(4-2.1x^2+\frac{x^4}{3}\right)x^2+xy+(-4+4y^2)y^2\right]$	2
F10: Shekel's foxholes	$f_{10}(x,y) = 500 - \frac{1}{0.002+\sum_{i=0}^{24}\frac{1}{1+i+(x-a(i))^6+(y-b(i))^6}}$	1

3. EXPERIMENTS

In this section, the performance of the proposed method is compared with some of the previously-introduced methods in the literature. Four variants of lbest PSO method [5], namely r3pso, r2pso, r3pso-lhc, and r2pso-lhc were applied in our comparisons with the proposed method. 10 of the well-known benchmark functions were used in these comparisons. The details of these multimodal benchmark functions are presented in Table 1. To achieve more stable results, we ran each method 50 times, and the averages of the results are presented in this section. The Success Rate measures was used in the experiment as the performance measure which describes the percentage an algorithm finds all peaks in a sequence of runs.

Table 2 displays the performance in terms of success rate. ε parameter in Table 2 refers to the desired accuracy (the maximum distance between obtained and desired peaks), and r stands for the minimum distance between two individuals to be considered as different peaks. As it can be observed in Table 2, the variants in the proposed method outperformed the previously-introduced methods in terms of success rate. The convergence behavior of the proposed method is depicted in Fig. 2.

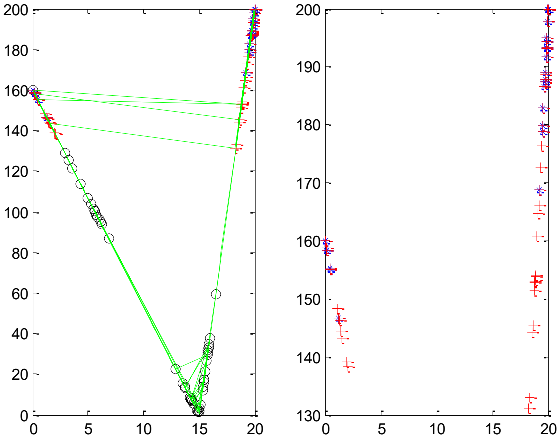
4. CONCLUSION

This paper proposes a niching algorithm utilizing a ring topology for multimodal optimization. An individual is considered in a separate niche along with its immediate neighbors on the left and right. Consequently, the proposed method introduces no additional parameters to the genetic algorithm (GA). The crossover operator is designed to construct offsprings using individuals within a niche, resulting in offsprings that belong to the same niche.The proposed method's ability in solving multimodal optimization was confirmed by the experimental results in terms of the success rate.

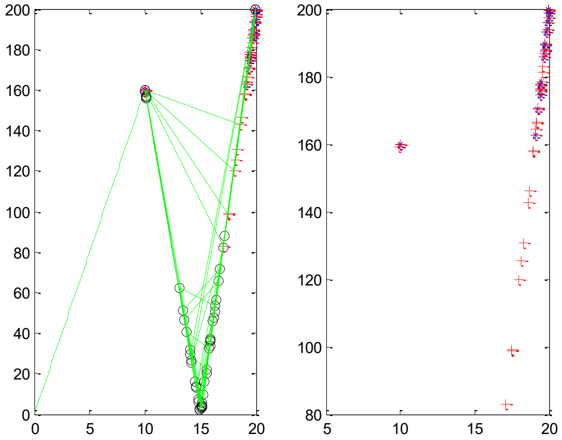
Table 2. Success Rate for the proposed method and the other methods.

function	***ε***	***r***	**R3GA**	**R2GA**	**R3GA-lhc**	**R2GA-lhc**	**R3PSO**	**R2PSO**	**R3PSO-lhc**	**R2PSO-lhc**
F1	0.1	0.5	1	1	0.86	1	1	0.98	0.78	0.94
F2	0.1	0.5	o.92	1	0.86	1	o.96	1	0.88	0.98
F3	5	0.5	1	1	0.98	0.96	0.96	1	0.96	0.96

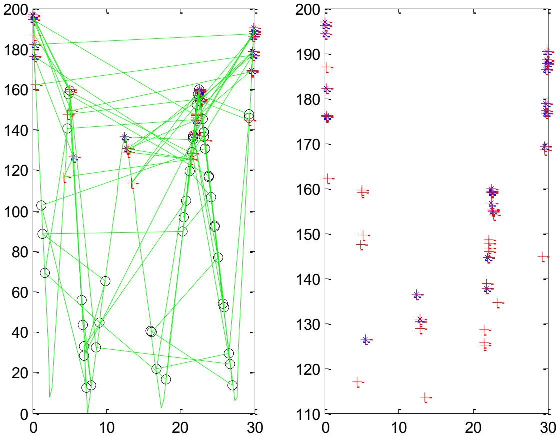
F4	0,01	0.01	1	1	1	1	1	1	1	1
F5	0.01	0.01	1	1	1	1	1	0.98	1	1
F6	0.01	0.01	1	1	1	1	0.98	0.98	1	1
F7	0.01	0.01	1	1	1	1	1	1	1	1
F8	0.1	0.5	0.62	0.88	0.94	0.98	0.74	0.92	0.98	1
F9	0.01	0.5	1	1	1	1	1	1	1	1
F10	0.01	0.5	1	1	0.80	0.82	1	1	0.78	0.72



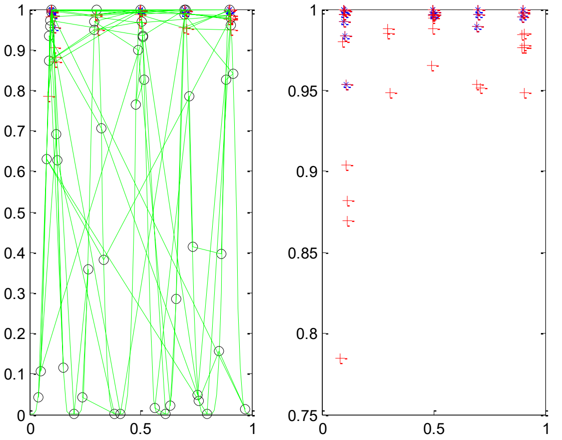
(a). F1



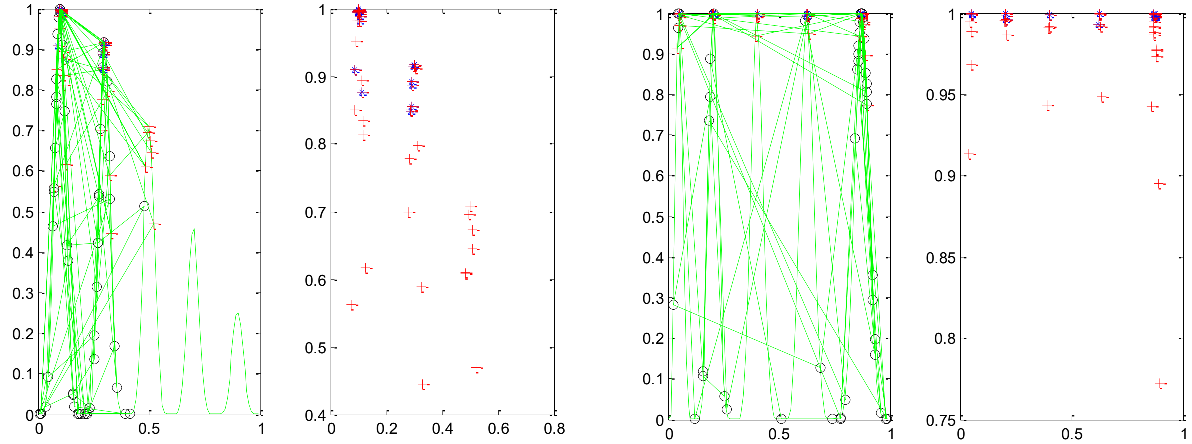
(b). F2



(c). F3



(d).F4



(e). F5

(f). F6

Fig. 2. r3GA with population size of 50 after 100 iterations (right plot), and their corresponding index position (left plot) for some of the benchmarks (F1-F6).

CONFLICT OF INTEREST

There is not any conflict of interest in this manuscript.

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