



Dynamic Modeling of Trains for Application in Railway Transportation Automation Systems

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ARTICLE INFO	ABSTRACT
<p>Article History: Received 11 December 2023 Received in revised form 16 February 2024 Accepted 9 March 2024 Available online 12 March 2024</p>	<p>The current railway transportation industry utilizes automation across various domains such as safety, stability, motion control, and train scheduling to optimize the use of railway resources and facilities. A prerequisite for these automation systems is the ability to predict and simulate train motion, which requires accurate train motion modeling. Factors such as air friction, the complexity of rail routes, the interaction forces between wagons, dynamics of force generation in actuators, and mechanical part friction result in nonlinear equations for train motion dynamics. Experimental values for some of these factors are uncertain due to wear and structural changes in components, while others are unmeasurable, complicating control conditions. This paper describes multi-particle and single-particle dynamic models of train motion. For practical operation, time-varying or unknown parameters in these equations are identified using a recursive least squares algorithm, and the estimated values are applied in a sliding mode control signal to compensate for baseline resistance and route disturbances. The designed sliding mode controller at the core of this system mitigates the effects of uncertainties and accurately tracks the desired speed-location profile. Simulation results presented in this paper demonstrate precise parameter estimation along with favorable tracking outcomes for the speed-location characteristic.</p>
<p>Keywords: Automation, Railway Transportation, Identification, Recursive Least Squares Algorithm, Sliding Mode Control</p>	

1. INTRODUCTION

Automation in the railway transportation industry has become an inevitable necessity, reducing accidents caused by human operator errors, such as train collisions or derailments, maintenance and wear-and-tear costs, fuel consumption, delays in train schedules, and optimizing the control of level crossings. It also improves accident reduction and management, emergency handling, and the efficient transportation of passengers and goods. The objectives of automation in this industry can be categorized into three groups: safety, efficiency, and management.

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These goals are achieved through automatic train protection (ATP), automatic train operation (ATO), and automatic train control (ATC) systems.

The ATP system monitors train speeds, ensuring they do not exceed predefined limits for the current block. The ATC system manages rail traffic schedules to optimize track utilization and transmits the necessary movement authorities to trains. The ATO system regulates the traction and braking forces to guide trains between two predefined stopping points according to a desired speed–position profile.

The schedules set by the ATC system and the desired speed–position profiles in the ATO system must be practically executable. Their design takes into account factors such as train density on the tracks, safe spacing between trains, time required for refueling, loading, dispatching, and overtaking, speed restrictions along the route, track parameters (gradient, curvature, tunnels, etc.), and train characteristics (weight, length, class, actuator limitations, etc.). The ATP system also relies on some of these features to predict train speeds and determine the required protective actions (e.g., braking to reduce speed or stop, or allowing the train to proceed without intervention). Consequently, all three systems require accurate predictions of train movement, or in other words, train motion simulations, to achieve their objectives. However, the complex dynamics of trains make modeling and controller design challenging.

A train consists of several wagons coupled with one or more locomotives, where traction and braking forces are not uniformly applied across all wagons, and the coupling equipment exhibits nonlinear behavior [1][2]. Parameters used in train modeling, derived from empirical measurements under ideal conditions, are influenced by environmental factors, wear, or component replacement [3]. Along the route, strong winds, tunnels, curves, gradients, and misalignment of the rails with the train's velocity vector can exert additional forces on the flanges. Some of these factors can be expressed mathematically, but their computations are often imprecise or overly complex, making their use impractical [3][4].

In early designs of ATO systems and their required controllers, simplified linearized train models and PID controllers were used. However, the approximations employed in such models resulted in severe control signal fluctuations to compensate for tracking errors, leading to increased component wear and passenger discomfort [5]. In subsequent designs, linear train modeling was abandoned in favor of two control approaches. The first involves control methods that do not directly depend on train dynamics and treat the system as a black box, such as nonlinear fuzzy algorithms or neural networks [6][7]. Reference [8] avoids dynamic train modeling altogether and proposes a fuzzy controller based on the train operator's experience in stopping the train. Reference [9] employs fuzzy logic in neural network learning to train the train's operation in different motion phases (acceleration, cruising, coasting, and stopping), while reference [10] uses iterative learning to interpret and track desired speed profiles accurately. Reference [11] employs offline data mining and analysis of previous stops to derive train motion characteristics and subsequently adjusts parameters using online learning.

These algorithms typically require a training phase, which delays real-time operation and necessitates repeated training for each train to improve modeling accuracy. Additionally, slow convergence rates and large data requirements are major drawbacks of these control methods.

In other train motion modeling designs for controller implementation, Newton's laws and dynamic motion equations are employed to mathematically model the system as a nonlinear one. Controllers based on such models often use sliding mode or adaptive control approaches due to their higher adaptability to parameter variations and disturbances. Examples include terminal sliding mode control or differential equations derived from the Lyapunov function derivative for parameter adaptation [12][13], integrating terminal sliding mode control with neural networks to learn the impact of route disturbances and modeling errors [3], and combining fuzzy control with sliding mode control [1].

In this paper, nonlinear single- and multi-particle train dynamics models are derived. The parameters in these models, including track parameters and basic train resistance, are identified using the recursive least squares method and are provided to the controller. Since the recursive least squares method performs online identification, parameter variations are also tracked in real time, preventing tracking errors in the controller.

The remainder of this paper is organized as follows: Section 2 reviews and presents the dynamic equations governing train motion. Section 3 outlines the theory of adaptive system identification using the recursive least squares method and applies it to train dynamics. Section 4 designs the required ATO system controller using nonlinear sliding mode control methods. Section 5 presents simulation results along with their analyses, and Section 6 concludes the paper.

2. TRAIN DYNAMICS MODELING

To model the dynamics of train motion, it is necessary to understand its components and the forces acting upon them. As described in the introduction, a train consists of one or more locomotives connected to freight or passenger wagons. These wagons are composed of chassis and body structures, movement systems, braking systems, coupling devices, and shock absorbers (buffers) [14]. The wagon chassis is responsible for bearing and distributing the load, while the body structure ensures passenger safety and comfort.

The movement system is categorized into two types: bogie-equipped and non-bogie systems. It includes components such as the chassis or frame, bogie cradle, wheels and axles, primary and secondary suspension systems, braking system, bearing boxes and fasteners, cradle bowl or plate, and hydraulic shock absorbers.

The braking systems of the wagons, which are responsible for reducing the train's speed, are available in various types, including dynamic, pneumatic, electro-pneumatic, and others. These systems can be approximated using a first-order transfer function with a delay component.

$$G_b(s) = \frac{e^{-Ts}}{\tau s + 1} \quad (1)$$

In the transfer function mentioned above, T represents the dynamic delay of the braking system, while τ denotes the transfer function coefficient [15]. Buffers and couplings are used at the connection points between two wagons, serving the purpose of damping the applied shocks and significantly preventing the train from experiencing sudden jolts.

The operation of the coupling system components can generally be modeled using the dynamics of a nonlinear spring-damper system as follows:

$$f_k(\delta x) = k \cdot \delta x \quad (2)$$

$$f_b(\delta \dot{x}) = b \cdot \delta \dot{x} \quad (3)$$

The coupling system can be modeled such that k represents the spring stiffness coefficient, δx indicates the relative displacement between two wagons, f_k is the restoring force generated by the spring, $\delta \dot{x}$ denotes the relative velocity between adjacent wagons, b is the damping coefficient of the damper, and f_b is the restoring force produced by the damper.

For a real spring, k is not constant but instead varies nonlinearly with the relative displacement, expressed as:

$$k = k_0(1 + \varepsilon \cdot \delta x^2) \quad (4)$$

where ε is the nonlinearity coefficient, and k_0 is the stiffness coefficient of a linear spring. As can be inferred from this equation, for a linear spring, $\varepsilon = 0$, and the worst-case scenario in train cruise control occurs when $\varepsilon < 0$ [16].

The forces affecting the motion of a train are divided into two categories: *tractive forces* (produced by the generator and motors) and resistive forces. The resistive forces include the force from the braking actuators, bearing forces, friction from flange wear on the rail, air resistance, energy losses, and others. Except for the braking force, which is controlled by the controller, the remaining resistive forces are classified into two types: forces that exist during motion on a straight and level track, typically arising from the internal dynamics of the train, and resistive

forces encountered during motion on a real-world track, which generally have an external source. The first type of resistance is called basic resistance, and the second is referred to as additional resistance.

All of the forces contributing to basic resistance are related to the train's speed in three ways. The first type appears as a constant coefficient in the train's dynamic equation and does not depend on speed, such as mechanical resistances inside the train components or energy loss during transmission. The second type, such as flange wear on the rail, is proportional to the speed of the train. The third type is proportional to the square of the speed, typically representing aerodynamic resistance. The total basic resistance, resulting from the sum of these three resistances, is denoted as $f_1(\dot{x}(t))$ or $f_1(v(t))$, and is expressed as:

$$f_1(v(t)) = a_0(t) + a_1(t)v(t) + a_2(t)v^2(t) \quad (5)$$

It is expressed as follows, where $x(t)$ is the displacement of the train along the track, and $a_i(t)$ ($i = 0,1,2$) are the coefficients of the train's basic resistance, which vary over time due to wear of components or changes in operational conditions [3]. The effective forces present in the additional resistance arise from three main factors: tunnel resistance R_T , curve resistance R_C , and track gradient resistance R_G , and are represented by $f_2(t)$.

$$f_2(t) = R_C + R_G + R_T = \frac{g \cdot m \cdot k_C}{L_C} + m \cdot g \cdot \sin\alpha + 0.00013 \cdot L_{TU} \quad (6)$$

In this equation, α represents the track gradient in degrees, g is the gravitational acceleration, m is the train's mass in kilograms, k_C is the empirical coefficient of twist, L_C is the curve radius, and L_{TU} is the tunnel length in meters. Although other factors, such as wind, rail wetness or dryness, etc., also affect the train's additional resistance, they are omitted here for simplicity [3], [17], and [18].

The final point in modeling the train's motion concerns the required dimensions. In a railway track, the train moves in three-dimensional space, where forward and backward motion, entering curves, or moving on a gradient, alters its longitudinal, lateral, and vertical coordinates. However, from the control perspective of automatic train systems, the critical factor is the displacement of the train along the railway path. Therefore, for modeling the train's motion, a one-dimensional space can be considered, where the variable represents the displacement of the train relative to the departure station.

Using the forces defined above and according to Newton's laws, which state that the sum of all forces acting on a body equals its mass times acceleration, the train can be modeled in two ways. In the first case, each wagon of the train is considered a rigid body that can exert force on its neighboring wagons. In this case, each wagon has specific parameters such as position, velocity, weight, rolling resistance coefficients (a_0 and a_1), control force, and each connection element has its own damping coefficient and spring stiffness. The majority of the aerodynamic resistance force in this case is applied only to the first wagon. The spring and damper forces act as resistance forces for the first wagon and as traction forces for the last wagon, but for the wagons in between, the forces will appear as traction from the front and resistance from the rear. As a result, the dynamic equations for these three parts are formulated as follows:

$$m_1 \ddot{x}_1 = u_1 - k_1(x_1 - x_2) - b_1(\dot{x}_1 - \dot{x}_2) - (a_0 + a_1 \dot{x}_1)m_1 - a_2 \dot{x}_1^2 \sum_{i=1}^n m_i - f_2 \quad (7)$$

$$m_i \ddot{x}_i = u_i - k_i(x_i - x_{i-1}) - b_i(\dot{x}_i - \dot{x}_{i-1}) - k_{i+1}(x_i - x_{i+1}) - b_{i+1}(\dot{x}_i - \dot{x}_{i+1}) - (a_0 + a_1 \dot{x}_i)m_i \quad (8)$$

$$m_n \ddot{x}_n = u_n - k_{n-1}(x_n - x_{n-1}) - b_{n-1}(\dot{x}_n - \dot{x}_{n-1}) - (a_0 + a_1 \dot{x}_n)m_n \quad (9)$$

are written as follows: (7) is the dynamic equation for the locomotive, (9) is the dynamic equation for the last wagon, and (8) is the dynamic equation for the intermediate wagons. For simplicity, it is assumed that the additional resistance caused by the track is only applied to the first wagon in the form of f_2 . In these equations, n is the number of wagons in the train set, i is the wagon number, and $i = 2,3, \dots, n - 1$, m_i is the mass of the i -th wagon, k_i is the

spring stiffness coefficient between the i -th and $i+1$ -th wagons, b_i is the damping coefficient between the i -th and $i+1$ -th wagons, u_i is the braking or traction force of the i -th wagon, and x_i is the longitudinal position of the i -th wagon. This type of modeling provides high accuracy; however, the number of sensors required for the controller and the number of unknown variables are substantial [16].

In the second case of the train's dynamic modeling, based on Newton's law of action and reaction between two bodies, the sum of the forces exerted by the coupling equipment is considered nearly zero, and the entire train is treated as a rigid body. In this case, the total forces applied to the train are expressed as...

$$m\ddot{x}(t) = u(t) - f_1(\dot{x}(t)) - f_2(t) \tag{10}$$

They are expressed as follows, where m is the mass of the train in kilograms (kg), $x(t)$ is the position of the train in meters (m), $u(t)$, $f_1(\dot{x}(t))$ and $f_2(t)$ represent the control signal, the base resistance, and the disturbances applied to the system in Newtons (N), respectively [19]. Despite its simplicity, the single-particle model demonstrated here has high efficiency in the design of rail transport system controls, and for this reason, the description in equation (10) will be used in the continuation of this paper to express the train's dynamics.

3. DESIGN OF THE RECURSIVE LEAST SQUARES ESTIMATOR FOR IDENTIFYING THE UNKNOWN PARAMETERS OF THE TRAIN DYNAMICS

The parameters m and $a_i(t)$ ($i = 0,1,2$) in equation (10) are variables that have specific values for each wagon and each train set. Additionally, the value of f_2 is determined based on the specific railway line. Therefore, a method must be adopted to identify, adapt, or robustly control the system in response to these changes. In this paper, assuming the approximate mass of the train is known, the remaining parameters are estimated using the Recursive Least Squares (RLS) algorithm to reduce the uncertainties in the system as much as possible.

The least squares method states that the unknown parameters in a system should be selected in such a way that they minimize the sum of the squared errors of the difference between the system's output and the value computed by the estimated parameters. According to reference [20], for a discrete-time system model in the form of equation (11), the objective function to minimize, $V(\theta, k)$, is written as equation (12), where the values $\varphi^T(i) = [\varphi_1(i); \varphi_2(i); \dots; \varphi_n(i)]$, $\hat{y}(i)$, $y(i)$, k and $\theta^T = [\theta_1; \theta_2; \dots; \theta_n]$ represent the regression vector, estimated parameters, system output at the i -th sampling instant, the number of sampling iterations, and the unknown parameter vector, respectively.

$$y(i) = \varphi^T(i)\theta = \varphi_1(i)\theta_1 + \dots + \varphi_n(i)\theta_n \tag{11}$$

$$V(\theta, k) = \frac{1}{2} \sum_{i=1}^k (y(i) - \varphi^T(i)\hat{\theta})^2 \tag{12}$$

To find the minimum value of the cost function, the derivative of equation (12) with respect to $\hat{\theta}$ is taken, and the resulting equation is set equal to zero. Finally, equation (13) is derived to calculate the unknown parameters, where:

$\Phi^T = [\varphi^T(1), \varphi^T(2), \dots, \varphi^T(N)]$ and $Y^T = [y(1), y(2), \dots, y(N)]$ and This leads to the estimation of the unknown parameters as shown in [20].

$$\hat{\theta} = (\Phi^T \Phi)^{-1} \Phi^T Y \tag{13}$$

The condition for using the above equation is that the matrix $\Phi^T \Phi$ must be non-singular, which means that the columns of Φ are linearly independent. As time progresses and the number of sampling times increases, the dimensions of the matrix Φ grow, resulting in a heavy computational burden on the system. To address this issue, by defining the matrix $p(t)$ as shown in equation (14), equation (13) is rewritten, enabling the computations to be performed recursively using the most recent values of the variables [20].

$$p(t) = \left(\sum_{i=1}^t \varphi(i)\varphi^T(i) \right)^{-1} \tag{14}$$

$$K(t) = p(t-1)\varphi(t)(I + \varphi^{-1}(t)p(t-1)\varphi(t))^{-1} \tag{15}$$

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)(y(t) - \varphi(t)^T\hat{\theta}(t-1)) \tag{16}$$

$$p(t) = (I - K(t)\varphi(t)^T)p(t-1) \tag{17}$$

According to equation (13), the number of sampling times of the system must be at least equal to the number of unknown parameters. However, since using the original equations from the first step is preferable, the matrix $p(t)$ is considered as shown in equation (18), where $p(t_0)$ is an arbitrary large matrix [20].

$$p(t) = (p^{-1}(t_0) + \phi_{(t)}^T\phi_{(t)})^{-1} \tag{18}$$

To account for slow variations of parameters over time, by considering a real number $0 < \gamma \leq 1$ as the exponential forgetting rate, equations (15) and (17) are written as follows to increase the influence of new samples in the current computation relative to previous samples [20]:

$$K(t) = p(t-1)\varphi(t) \times (\lambda + \varphi^{-1}(t)p(t-1)\varphi(t))^{-1} \tag{19}$$

$$p(t) = (I - K(t)\varphi(t)^T)p(t-1)/\gamma \tag{20}$$

In the automatic train control problem, to estimate the values of a_i and f_2 in equation (10), the system's dynamic equation must be rewritten in discrete time. Assuming a time constant $\tau = 0.1(s)$, the sequence $k=0,1,2,\dots,N$, and the first-order approximation $\dot{v}_{(t)} = (v(\tau k) - v(\tau(k-1)))/\tau$, the time parameter is discretized as $t = k\tau$. The discrete-time dynamic equation of the train's motion is given by equation (21), where, for simplicity, the τ -related coefficients are not explicitly written in the sampling.

$$\frac{m}{\tau}(v(k) - v(k-1)) = u(k-1) - a_0(k-1) - a_1(k-1)v(k-1) - a_2(k-1)v^2(k-1) - f_2(k-1) \tag{21}$$

By rewriting equation (21) as equation (22), the recursive least squares method can be applied as in equation (11), where in equation (123), the vector φ is defined, and in equation (24), the unknown parameters θ are defined.

$$y(k) = \frac{m}{\tau}(v(k) - v(k-1)) - u(k-1) = \varphi^T\theta \tag{22}$$

$$\varphi(k-1) = \begin{bmatrix} -1 \\ -v(k-1) \\ -v^2(k-1) \end{bmatrix} \tag{23}$$

$$\theta = \begin{bmatrix} f_2(k-1) + a_0(k-1) \\ a_1(k-1) \\ a_2(k-1) \end{bmatrix} \tag{24}$$

Another method for fitting equation (21) with equation (11) involves considering the output as equation (22), where the vector φ is taken as a scalar value of negative one, and the unknown parameters are represented as scalars in equation (25).

$$\theta = -a_0(k-1) - a_1(k-1)v(k-1) - a_2(k-1)v^2(k-1) - f_2(k-1) \quad (25)$$

By applying the recursive least squares estimation method to equations (25) or (24), the parameters \hat{a}_i ($i = 0,1,2$) and \hat{f}_2 will be calculated, and the mathematical model (11) will be derived with specified parameters.

4. SLIDING MODE CONTROLLER DESIGN

In order to utilize the models developed for train motion simulation, a train guidance controller needs to take over the role of the human operator. In this paper, among the available controllers, the sliding mode control method is used, which is a robust approach. It operates on the principle that controlling a first-order system is simpler than controlling a system of order n . In sliding mode control, the state space is divided into two regions: the sliding surface and the region outside of it. When variables traverse a path in the state space and reach the sliding surface, a discrete change occurs, and from there, the system is directed to the desired coordinates by the surface [21].

To apply the sliding mode controller to automatic train guidance, consider the single-input dynamics in equation (10), where the function $f_1(v(t))$ is nonlinear with variable coefficients, and the function $f_2(t)$ is an unknown function. By considering the position of the train, $x(t)$, in equation (10) as the state variable, the control objective is defined as tracking the time-varying desired vector $x_d(t)$. To transform the tracking problem into a regression problem, the tracking error is defined as a new variable in the form of equation (26).

$$e(t) = x(t) - x_d(t) \quad (26)$$

As a result, the control objective is formulated as driving the tracking error to zero and keeping it at zero, as described in equation (26). Subsequently, the sliding surface is defined as:

$$s(t) = \dot{e}(t) + \lambda e(t) \quad (27)$$

It is defined, where λ is a free design parameter with a positive real value. If the sliding surface ss can be maintained at zero, the differential equation $\dot{e}(t) = -\lambda e(t)$ is derived from (27), which has the solution $e(t) = e(0)exp(-\lambda t)$. This response indicates that for a positive value of λ , the position error will tend towards zero, and the control objective will be achieved in a finite time. In order to satisfy the condition $s = 0$ and track the signal $x_d(t)$, both ss and all of its derivatives must be zero. Consequently, the derivative of (27) is computed and set equal to zero.

$$\dot{s} = \ddot{e}(t) + \lambda \dot{e}(t) = (\ddot{x}(t) - \ddot{x}_d(t)) + \lambda \dot{e}(t) = 0 \quad (28)$$

By substituting (10) into the simplified equation (28), expression (29) is obtained, which represents the relationship between the sliding surface and the control signal.

$$\dot{s} = \frac{1}{m}(u(t) - f_2 - a_0 - a_1 v(t) - a_2 v^2(t)) - \ddot{x}_d(t) + \lambda \dot{e}(t) = 0 \quad (29)$$

The best continuous estimate of the control signal that can ensure the establishment of (29) is given by (30).

$$u_{eq}(t) = m(\ddot{x}_d(t) - \lambda \dot{e}(t)) + \hat{f}_2 + \hat{a}_0 + \hat{a}_1 v(t) + \hat{a}_2 v^2(t) \quad (30)$$

The parameters \hat{a}_i ($i = 0,1,2$) and \hat{f}_2 are the estimated values of the coefficients a_i ($i = 0,1,2$) and f_2 obtained using the recursive least squares method in equation (10). By establishing the relation (28), remaining on the sliding surface is guaranteed. However, to move towards the surface when we are outside of it, the sliding condition needs to be written as:

$$\frac{1}{2} \left(\frac{d}{dt} s^2 \right) \leq -\eta |s| \tag{31}$$

It should be established that η is a positive value. In this case, the term u_{sw} , according to the definition in (32), needs to be added to the control signal.

$$u_{sw}(t) = -\rho \text{sat}(s) \tag{32}$$

$$\text{sat}(s) = \begin{cases} 1 & \text{if } s > \delta \\ \frac{s}{\delta} & \text{if } -\delta \leq s \leq \delta \\ -1 & \text{if } s < -\delta \end{cases} \tag{33}$$

The control signal in its final form will be as shown in (34), which is the sum of u_{eq} and u_{sw} .

$$u(t) = m(\ddot{x}_d(t) - \lambda \dot{e}(t)) + \hat{f}_2 + \hat{a}_0 + \hat{a}_1 v(t) + \hat{a}_2 v^2(t) - \rho \text{sat}(s) \tag{34}$$

5. SIMULATION

To assess the accuracy of the parameter estimation, a Simulink simulation environment in MATLAB version 2020b has been used. According to the design assumptions, in all simulations, the values of $u(t)$, $x(t)$, $v(t)$, $x_d(t)$, $v_d(t)$, and their derivatives are either specified or computable. The train's mass is set to 350 tons, and all forces are considered in kilonewtons. Additionally, a saturation level of 250 kilonewtons for the control signal is assumed [9]. The desired speed-position characteristic in this simulation is shown in Figure 1. The maximum train speed in Figure 2 is 83 m/s, approximately equivalent to 300 km/h, and the distance covered is 74,700 meters.

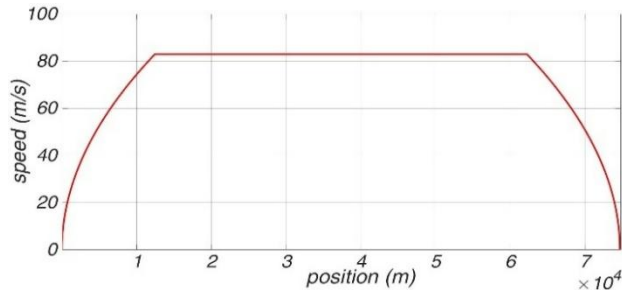


Fig. 1. Desired Speed-Position Profile

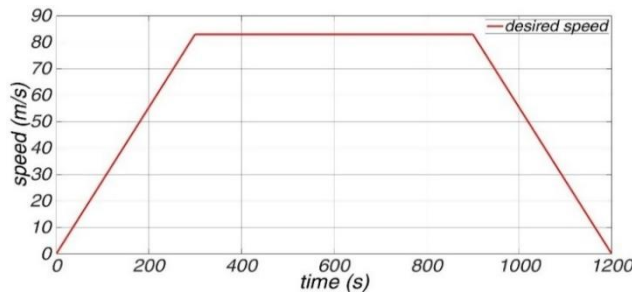


Fig. 2. Desired Speed-Time Profile

The disturbance assumed in the simulations for the parameter $f_2(t)$ is only caused by the slope of the railway track, as shown in Table 1, which applies a force equivalent to that in Figure 3 to the system [1].

Table 1. Track Slope Parameters as a Function of Train Location

Parameter (%) / Slope	Location Range (m)	Value
-10%	0 - 5000	-10
9%	5000 - 10000	9
-8%	10000 - 25000	-8
-7%	25000 - 42000	-7
20%	42000 - 50000	20
-14%	50000 - 62000	-14
2%	62000 - 70000	2
15%	70000 - 75000	15

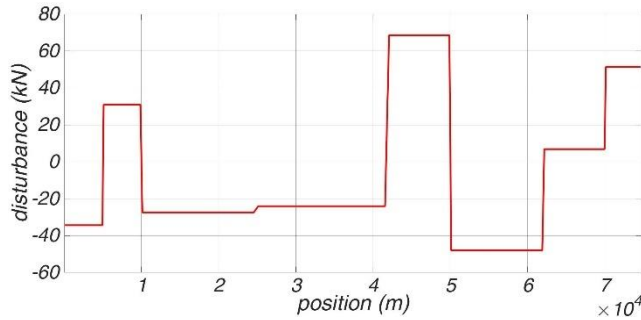


Fig. 3. Disturbance Force from Track Slope

The values of the coefficients $a_i (i = 0,1,2)$ in the base resistance are defined according to equation (35), and they will vary by $\pm 10\%$ over the course of the simulation period (1200 seconds) [12].

$$\begin{aligned}
 a_0 &= 2.09 + 0.2 \sin(0.0052 \times t) \\
 a_1 &= 0.039 + 0.004 \sin(0.0052 \times t) \\
 a_2 &= 0.000675 + 0.0000067 \sin(0.0052 \times t)
 \end{aligned} \tag{35}$$

For the controller, the sliding mode control law (34) will be used, and for the identification of the base resistance parameters, definitions (22) to (24) and equations (19), (16), and (20) will be applied. The control parameter λ is set to 0.5 through trial and error, ρ is set to 180, the parameter δ in (33) is set to 5, and the initial values used in the identification method are given by (36).

$$p_0 = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.001 \end{bmatrix}, \theta_0 = 0 \tag{36}$$

Since the value of the additional resistance changes suddenly according to Figure 3, the matrix p_{k-1} needs to be reset in the least squares identification method in order to adapt to abrupt changes. In the simulation below, by resetting the matrix p_{k-1} to the value p_0 every 30 seconds, the estimates of the parameters a_1 and f_2 are plotted. According to the definition in (24), in the uppermost plot, the expression $a_0 + f_2$ is estimated, and for simplicity, the value θ_1 is used in the figures instead.

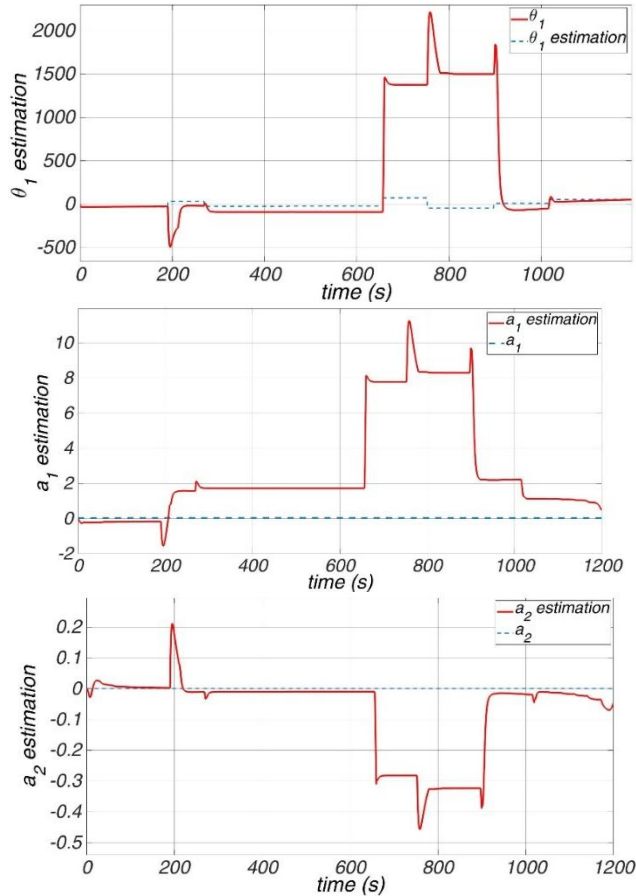


Fig. 4. Estimation of parameters \mathbf{a}_i and \mathbf{f}_2 with a time-resetting estimator.

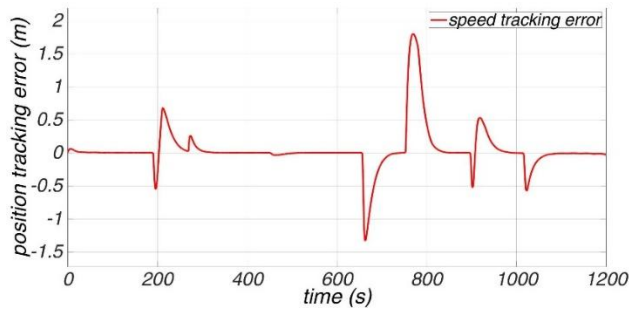


Fig. 5. Location tracking error with time-resetting estimator and controller (34).

As shown in Figure 4, there is significant error in the estimation of all three parameters a_i and f_2 , and the location tracking error in Figure 5 exceeds one meter. As a result, the time-resetting method for estimation will not be suitable. To eliminate this issue, the method of resetting the matrix p_{k-1} spatially can be used, which can be implemented in several ways for trains. Methods such as using a camera for image processing in front of the train to detect changes in gradient or curvature, using an offline table for the location of gradient and curvature, comparing the train's location with it using positioning systems like GPS and informing the train via radio communications, or using magnetic warning signs before entering a curve or gradient, are some of the methods that can facilitate spatial resetting of the matrix p_{k-1} . The following figures show the results of simulations performed with spatial resetting estimation.

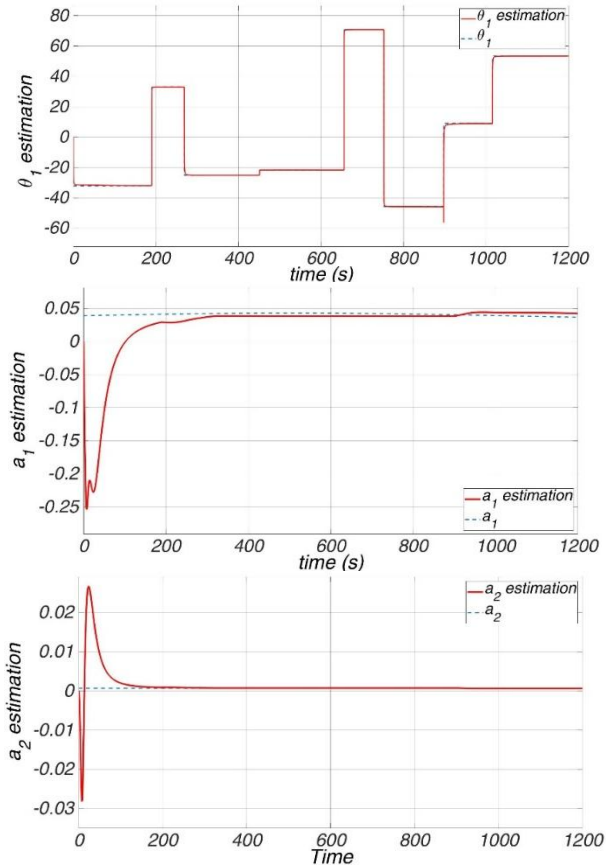


Fig. 6. Estimation of Parameters a_i and f_2 Using the Spatial Reset Identifier

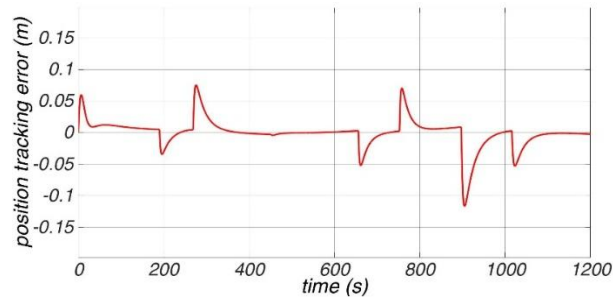


Fig. 7. Location tracking error with spatial-resetting estimator and controller (34).

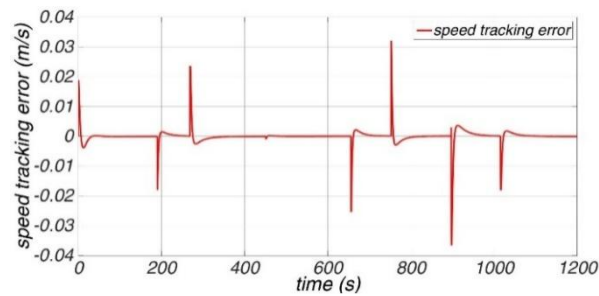


Fig. 8. Speed tracking error with spatial-resetting estimator and controller (34).

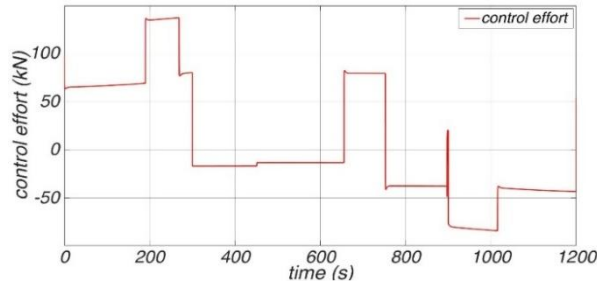


Fig. 9. Control signal generated by the controller (34) in the spatial-resetting estimator.

Figure 6 demonstrates that spatial-resetting can effectively identify the parameters a_i and f_2 . According to this figure, a maximum of 200 seconds is required to obtain accurate estimates of the parameters a_1 and a_2 , which is a reasonable time given the train's acceleration procedure, as these parameters only influence the dynamics at high speeds. The control law (34) successfully achieves position tracking with an accuracy of 0.12 meters, as shown in Figure 7, and speed tracking with an accuracy of 0.04 m/s, as shown in Figure 8. The instantaneous errors observed in Figure 7 are due to the time delay between the introduction of disturbances into the system and their elimination by the controller. The control signal generated in this case is shown in Figure 9 and remains entirely within the permissible range for the actuators. The jumps observed in this figure are due to the phase shifts in the train's motion (from acceleration to cruising and cruising to braking) and the slope of the track.

If the identification of the parameters a_i and f_2 is not intended separately, Equation (25) can be used to identify the unknown system parameters at once. In this case, due to the relatively large time scale of the track's parameter changes compared to the number of data points considered in the recursive calculations, only an exponential forgetting factor of 0.95 can be used to match the new values. The advantage of this approach over the third case is that there is no need to alter the physical structure or add new hardware to the railway line or train. As shown in Figure 10, the estimate obtained in this case is more accurate, leading to better control performance. For simplicity, the estimated parameter in Equation (25) is represented as θ_0 in the figures of this section of the simulation.

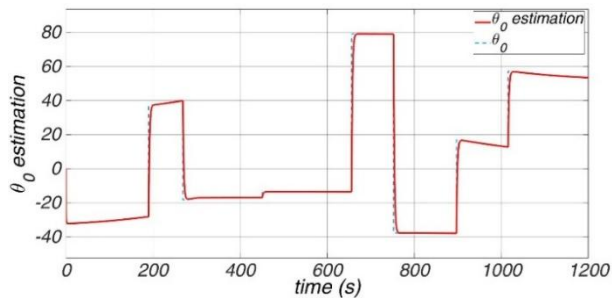


Fig. 10. Synchronized estimation of the parameters a_i and f_2 through Equation (25).

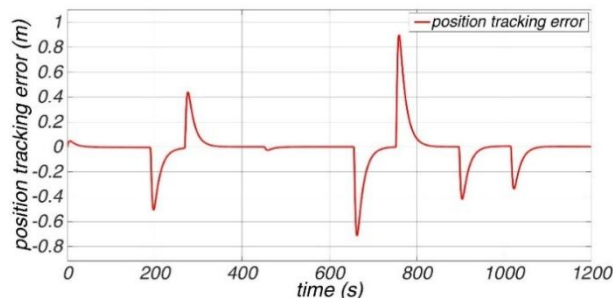


Fig. 11. Tracking error in position with the estimator using Equation (25).

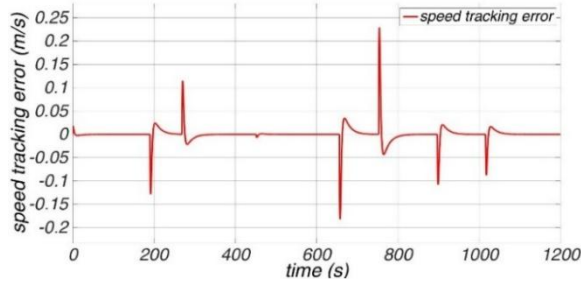


Fig. 12. Tracking error in velocity with the estimator using Equation (25).

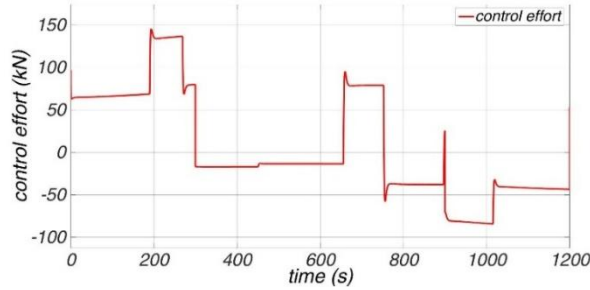


Fig. 13. Control signal generated by the controller (34) in the estimator using Equation (25).

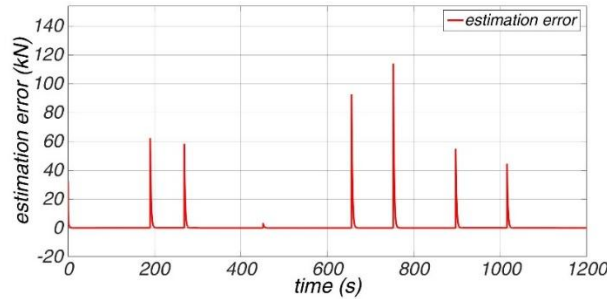


Fig. 14. Estimation error in the controller (34) using the estimator with Equation (25).

The estimation shown in Figure 10 provides a very accurate estimate of the sum of the unknown values in the train's dynamic equation, which converges within 30 seconds. This precise estimation allows the controller to achieve highly accurate tracking of the desired position and velocity characteristics, as shown in Figures 11 and 12, and ensures precise stopping at the station. The maximum position and velocity tracking errors are 0.82 m and 0.22 m/s, respectively. The parking error in Figure 11 is 0.002 meters, indicating a very precise stop. The jump in the control signal shown in Figure 9 has been improved as seen in Figure 13, though its oscillations increase during the application of disturbances. To strike a balance between factors such as passenger comfort and component wear while maintaining accurate tracking of the desired characteristics, the design parameters can be fine-tuned. Figure 14 shows the error in estimating the system's unknown parameters. This figure demonstrates that the estimation is always convergent, and the largest error corresponds to the moment of the largest external disturbance applied to the system.

6. CONCLUSION

Given the importance of automation in the railway transportation industry and the need to simulate train motion for forecasting and planning railway lines, this paper describes both the single-particle and multi-particle dynamics of a train. In the single-particle equations, time-varying resistance coefficients and real disturbances from the railway track were used, while in the multi-particle dynamics, in addition to the aforementioned factors, the nonlinear dynamics of springs and dampers between the wagons were also discussed. The impact of both the acting and reacting forces of each component on the train's dynamics was examined.

Then, to utilize the extracted single-particle model, the identification of the required parameters was performed. According to the designs made in this paper, using the recursive least squares method, the time-varying coefficients of base resistance, along with disturbances from the track due to gradients or curves, are identified and used in the sliding mode control law. Through this approach, the designed controller is robust to system uncertainties and can, while accurately estimating the values of base resistance and track parameters, reduce tracking error to zero and ensure precise stopping at the station.

Declaration

We acknowledge that we used ChatGPT to enhance the academic writing of our manuscript while ensuring the originality and integrity of our work.

Transparency Statement

The data supporting this study are available upon reasonable request to the corresponding author, subject to ethical and confidentiality considerations.

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Declaration of Interest

The authors declare that they have no competing interests.

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