



Estimation of Angular Velocity and Position of a Permanent Magnet Synchronous Motor Using Discrete-Time Extended Kalman Filter and Hybrid Extended Kalman Filter

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ARTICLE INFO	ABSTRACT
<p>Article History: Received 25 December 2019 Received in revised form 15 January 2020 Accepted 8 March 2020 Available online 9 March 2020</p>	<p>This paper presents an analytical study of the Permanent Magnet Synchronous Motor (PMSM), focusing on the estimation of its rotor's angular position and velocity using nonlinear filtering techniques. Due to the inherent nonlinearity of the PMSM's state-space equations, conventional linear estimation methods are insufficient for accurate state estimation. In this work, the motor's armature current being a directly measurable quantity is utilized as the primary observable to estimate the system's internal states, specifically the rotor's angular position and angular velocity. Two advanced estimation algorithms are employed and compared: the Discrete Extended Kalman Filter (DEKF) and the Hybrid Extended Kalman Filter (HEKF). These algorithms are adapted to handle the nonlinear nature of the PMSM dynamics and to operate effectively within a discrete-time framework. Simulation results reveal that both filters are capable of estimating rotor dynamics with reasonable accuracy. However, the hybrid Kalman filter demonstrates superior performance in terms of estimation precision and convergence speed. The findings highlight the effectiveness of hybrid nonlinear filtering approaches in improving the observability and control performance of PMSM systems, particularly in sensorless or low-cost applications where direct measurement of rotor position is impractical.</p>
<p>Keywords: Discrete-Time Extended Kalman Filter, Hybrid Extended Kalman Filter, Motor State Estimation, Nonlinear Systems, Permanent Magnet Synchronous Motor.</p>	

1. INTRODUCTION

Since permanent magnet synchronous motors have simple structure, high power and output coefficients, and high power density, and since they are reliable for industrial drives, they are very attractive in the industry. In the permanent magnet synchronous motor drive system, angular velocity control is done using rotor's angular velocity and position data measured by shaft sensors like optical encoders, Hall-effect sensors, etc. But using these sensors increases the weight and cost of the system and decreases the overall stability and reliability of the whole drive

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system. Hence, so many researches have been held to achieve sensorless control of the permanent magnet synchronous motors.

Many methodologies have been developed to determine rotor’s angular position and velocity using the measurement of electric quantities without the need of mechanical sensors. In [1], rotor’s angular position is estimated using a sliding mode observer but this method is not robust. Also, motor’s angular velocity is estimated using back-ekf observer but this estimation method does not have acceptable performance for low angular velocities [2]. Another estimation method uses pulse injection but the drive output is low for this method [3]. Other methods are developed on the basis of state observers but linearizing the nonlinear equations describing drive behavior along the nominal trajectory does not guarantee the overall stability [4]. In [5], an approach based on sensorless control is presented. Its results are intelligent, its implementation is efficient to a certain extent, its sensitivity to motor parameters can have a great effect on the estimate, and this method needs initial settings to rotor. Eventually, extending this method for anisotropic permanent magnet synchronous motor may be difficult due to the increasing complexity of the motor model. Extended Kalman filters have found widespread applications in the field of position and velocity estimation of synchronous motor drives due to their ability to perform state estimation of nonlinear systems. In [6], an extended Kalman filter is implemented for permanent magnet synchronous motor using matrix converter. In [7], an extended Kalman filter algorithm for permanent magnet synchronous motor vector control optimal performance is presented. In [8], linear Kalman filter is designed and implemented using orthogonal output of the linear model of the permanent magnet synchronous motor. Also in [9] and [10], parallel reduced-order Kalman filter is implemented on Digital Signal Processor (DSP). Furthermore, in [11] and [12], reduced-order Kalman filter estimates angular velocity and position of permanent magnet synchronous motor using Field-Programmable Gate Array (FPGA).

In this paper we consider the continuous-time state equations of permanent magnet synchronous motor with sampling period of 1ms. Also, taking into consideration that the current passing through the armature is measurable, measurements include only armature windings current.

The paper is organized as follows. First, in sections 2 and 3, the dynamics of the permanent magnet synchronous motor is presented, then Kalman filter is briefly overviewed. In sections 4 and 5, Discrete-time Extended Kalman Filter (DEKF) and Hybrid Extended Kalman Filter (HEKF) algorithms for estimating position and angular velocity of permanent magnet synchronous motor are presented. Tuning filter parameters and simulation results of applying discrete-time extended Kalman filter and hybrid Kalman filter are shown in sections 6 and 7. Finally, the conclusion of the paper is presented.

2. DYNAMICS OF PERMANENT MAGNET SYNCHRONOUS MOTOR

The considered system is 3-phase permanent magnet synchronous motor having a permanent magnet installed on the motor and sinusoidal flux. By linear transformations, this motor is transferred into a motor with fixed stator $[\alpha, \beta]$ having currents i_α and i_β , rotor angular velocity ω_r , and rotor angular position θ_r as the state variables:

$$\frac{d}{dt}i_\alpha = -\frac{R_s}{L_s}i_\alpha + \frac{\lambda_f}{L_s}\omega_r \sin(\theta_r) + \frac{V_\alpha}{L_s} + w_1 \tag{1}$$

$$\frac{d}{dt}i_\beta = -\frac{R_s}{L_s}i_\beta - \frac{\lambda_f}{L_s}\omega_r \cos(\theta_r) + \frac{V_\beta}{L_s} + w_2 \tag{2}$$

$$\frac{d}{dt}\omega_r = \frac{3\lambda_f}{2J}(i_\beta \cos(\theta_r) - i_\alpha \sin(\theta_r)) - \frac{B}{J}\omega_r - \frac{T_L}{J} + w_3 \tag{3}$$

$$\frac{d}{dt}\theta_r = \omega_r + w_4 \tag{4}$$

Measurement equations are also as follows:

$$y = \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \quad (5)$$

In the above relations, i_α and i_β are the currents passing through the two windings of the motor, θ_r and ω_r are, respectively, the angular position and velocity of the rotor, R_s is stator resistance in each phase, L_s is stator inductance of the winding of each phase, λ_f is the permanent magnet flux, B is the viscous friction coefficient between motor shaft and load, J is the moment of inertia between motor shaft and load, $w = [w_1 \ w_2 \ w_3 \ w_4]^T$ is process noise, $v = [v_\alpha \ v_\beta]^T$ is measurement noise, and y is the measurement vector. The voltages V_α and V_β and the mean load torque T_L are the deterministic inputs to the system. The two components of voltage and current are measurable and can be determined from the phases of the stator by linear transformations as follows:

$$i_\alpha = \frac{2}{3} \left(i_a - \frac{i_b}{2} - \frac{i_c}{2} \right) \quad (6)$$

$$i_\beta = \frac{i_b - i_c}{\sqrt{3}} \quad (7)$$

Voltage equations are similar to Equation (3). In summary, the system is stimulated by stator voltages V_α and V_β , and the result is stator currents i_α and i_β . Also, because of the product of state variables ω_r , θ_r , i_α and i_β , the state space model (1) is nonlinear. The following figure shows a model of a 2-pole synchronous motor:

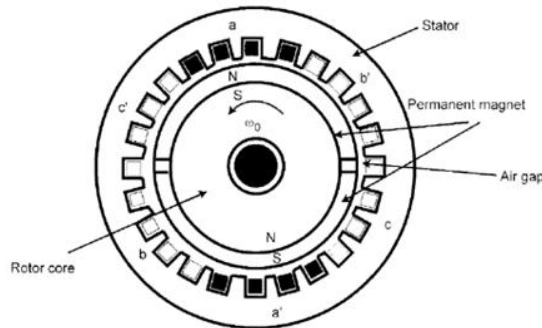


Fig. 1. 2-pole synchronous motor.

3. KALMAN FILTER OVERVIEW

Kalman filter is a set of mathematical relations providing state estimation of a system by minimizing mean squares error using a recursive algorithm. This filter is powerful in estimating past, present, and future states of the system.

Generally, Kalman filter is used to estimate a state vector of some discrete-time control process described by the following differential equations:

$$x_{k+1} = Ax_k + Bu_k + w_k \quad (8)$$

$$y_k = Cx_k + v_k \quad (9)$$

In the above equations, A, B, C are matrices, k is discrete-time index, x is the system state vector, u is the deterministic input to the system (known as the control signal), y is the measurements vector, and w, v are noises where w is the process noise and v is the measurement noise.

In the estimation problem we need to estimate the state vector that contains all system information. The main difficulty is that the state vector cannot be directly estimated but we can measure the vector y which is a function of the state vector disturbed by measurement noise. Therefore, to estimate the state vector we use the measurement

vector but this measurements are not totally reliable because they are always disturbed by noise. For this purpose, to improve the estimation, we use Kalman filter. Kalman filter not only uses measurements, but also uses system state equations and existing information in it. Kalman filter equations are the following:

$$K_{k+1} = P_k C^T [C P_k C^T + R]^{-1} \quad (10)$$

$$\hat{x}_{k+1} = [A \hat{x}_k + B u_k] + K_k [y_k - C \hat{x}_k] \quad (11)$$

$$P_{k+1} = [I - K_k C] P_k A^T + Q \quad (12)$$

In equation (10-12), \hat{x}_k is an estimate of x_k , K_k is a matrix called Kalman filter gain, P_k is the estimation error covariance matrix, Q is the process noise covariance matrix, and R is the measurement noise covariance matrix.

To initialize Kalman filter, we need to start with an initial estimate. Also we need an initial value of the covariance error matrix which represents the uncertainty of the initial estimate. The more accurate the initial estimate be, the less the covariance matrix would be and vice versa. Therefore, initial information will widely affect the performance of Kalman filter.

According to the above relations, Kalman filter is a linear filter and can be implemented for linear systems. Unfortunately, in practice linear systems do not exist unless, in some range of their workspace, they could be linear and only therein, linear filters like Kalman filter can be applied. Eventually, we always deal with systems that have linear behavior in a very small range of their workspace and Kalman filter cannot be applied to them globally. For this purpose, we need nonlinear filter.

Implementing nonlinear filters could be difficult and complex and it certainly could not be considered similar to linear methods. However, some nonlinear methods are developed. Some of these nonlinear methods are extended Kalman filter, unscented Kalman filter, particle filter, etc.

4. DISCRETE-TIME EXTENDED KALMAN FILTER AND HYBRID EXTENDED KALMAN FILTER

Kalman filter is a tool used to estimate the states of linear systems. However, when the considered system is nonlinear or the measurement relation is nonlinear, nonlinear filters are used. In this paper, because of the nonlinearity of the permanent magnet synchronous motor state equations, we use two nonlinear filters, discrete-time extended Kalman filter and hybrid extended Kalman filter, to estimate the states of this system. Therefore, the following sub-sections we present the algorithms of these two filters.

4.1. Discrete-time Extended Kalman Filter

Extended Kalman filter works like linearized Kalman filter with the difference that instead of the nominal trajectory (for systems that we do not have their nominal trajectory), the state estimate itself is used as the nominal trajectory. In another words, in the linearized Kalman filter, if we used the state estimate instead of the nominal trajectory, the resulting filter is extended Kalman filter. This method is an intelligent approach for estimating system states in which we use a nominal trajectory to estimate states, then we use that estimate as a nominal trajectory. By this substitution and by some mathematical operations and simplifications, extended Kalman filter algorithm would be as follows:

1. Equations describing system are as follows:

$$x_{k+1} = f_k(x_k, u_k) + w_k \quad (13)$$

$$y_k = h_k(x_k) + v_k \quad (14)$$

$$w_k \sim (0, Q_k) \quad (15)$$

$$v_k \sim (0, R_k) \quad (16)$$

where w_k is process noise and v_k is measurement noise.

2. For $k = 1, 2, 3, \dots$, we do the following:
 - a. Calculating the following partial derivative matrices:

$$A_k = f'_k(\hat{x}_k, u_k) \quad (17)$$

$$C_k = h'_k(\hat{x}_k) \quad (18)$$

Note that these derivatives are taken with respect to x_k .

- b. Using state and covariance propagation equations, we update the state estimate and error covariance:

$$\tilde{x}_{k+1} = f_k(\hat{x}_k, u_k) \quad (19)$$

$$\tilde{P}_{k+1} = A_k P_k A_k^T + Q \quad (20)$$

2. Now we update the state estimate and error covariance using Kalman filter relations as follows:

$$K_k = \tilde{P}_k C_k^T (C_k \tilde{P}_k C_k^T + R)^{-1} \quad (21)$$

$$\hat{x}_{k+1} = \tilde{x}_{k+1} + K_k [y_k - h(\hat{x}_k)] \quad (22)$$

$$P_{k+1} = (I - K_k C_k) \tilde{P}_k \quad (23)$$

4.2. Hybrid Extended Kalman Filter

Many of real engineering systems are expressed by continuous dynamics while the measurements are taken as discrete time measurements. In this sub-section we present the hybrid extended Kalman filter where the system is considered to be continuous and the measurement are considered to be discrete-time. This case is more familiar in practice. Hybrid extended Kalman filter can be summarized as follows:

1. Let the system be expressed using continuous-time dynamic equations and discrete-time measurements as follows:

$$\dot{x} = f(x, u, t) + w(t) \quad (24)$$

$$y_k = h_k(x_k) + v_k \quad (25)$$

$$w(t) \sim (0, Q) \quad (26)$$

$$v_k \sim (0, R_k) \quad (27)$$

2. For $k = 1, 2, 3, \dots$, we do the following:
 - a. Calculating the following partial derivative matrices:

$$A = f'(\hat{x}, u_k) \quad (28)$$

$$C_k = h'_k(\hat{x}_k) \quad (29)$$

- b. The integration of state estimate and error covariance from instant $(k-1)^+$ to instant k^- is given by:

$$\dot{\hat{x}} = f(\hat{x}, u, t) \quad (30)$$

$$\dot{P} = AP + PA^T + Q \quad (31)$$

In the integration process, the integration upper limit is considered to be $\hat{x} = \hat{x}_{k-1}^+$ and $P = P_{k-1}^+$, and the integration lower limit is considered to be $\hat{x} = \hat{x}_k^-$ and $P = P_k^-$.

3. Now we update the state estimate and error covariance using Kalman filter equations as follows:

$$K_k = \tilde{P}_k C_k^T (C_k \tilde{P}_k C_k^T + R)^{-1} \tag{32}$$

$$\hat{x}_{k+1} = \tilde{x}_{k+1} + K_k [y_k - h(\hat{x}_k)] \tag{33}$$

$$P_{k+1} = (I - K_k C_k) \tilde{P}_k \tag{34}$$

5. PERMANENT MAGNET SYNCHRONOUS MOTOR STATE ESTIMATION USING DISCRETE-TIME KALMAN FILTER AND HYBRID EXTENDED KALMAN FILTER

In this section, we estimate permanent magnet synchronous motor state variables using two estimation filters, discrete-time extended Kalman filter and hybrid extended Kalman filter, where the performance of the hybrid extended Kalman filter is much better.

5.1. Synchronous Motor State Estimation Using Discrete-Time Kalman Filter

To implement discrete-time extended Kalman filter for permanent magnet synchronous motor, first we have to define system states. Recalling equation (1) which are the equations of permanent magnet synchronous motor, the state variables are defined as follows:

$$x = \begin{bmatrix} i_\alpha \\ i_\beta \\ \omega_r \\ \theta_r \end{bmatrix} \tag{35}$$

Discretizing equation (1), system’s differential equations becomes as follows:

$$x_{k+1} = f(x_k, u_k) + w_k = x_k + \begin{bmatrix} \frac{-R_s x_k(1)}{L_s} + \frac{\lambda_f x_k(3) \sin x_k(4)}{L_s} + \frac{V_\alpha}{L_s} \\ \frac{-R_s x_k(2)}{L_s} - \frac{\lambda_f x_k(3) \cos x_k(4)}{L_s} + \frac{V_\beta}{L_s} \\ \frac{-3\lambda_f x_k(1) \sin x_k(4)}{2J} + \frac{3\lambda_f x_k(2) \cos x_k(4)}{2J} + \frac{-Bx_k(3)}{J} - \frac{T_L}{J} \\ x_k(3) \end{bmatrix} T_s + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} T_s \tag{36}$$

$$y_k = h(x_k) + v_k = \begin{bmatrix} x_k(1) \\ x_k(2) \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{37}$$

To use extended Kalman filter method, we have to differentiate $f(x_k, u_k)$ and $h(x_k)$ with respect to x_k which is a difficult task because the two matrices $f(x_k, u_k)$ and $h(x_k)$ are made of x_k vector. Here, we use the concept of differentiating functions to multiple variables. We want to differentiate some vector with respect to another vector. Therefore, we define these two vectors as follows:

$$f = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{38}$$

Now, the derivative of vector f with respect to vector x is obtained as follows:

$$\frac{\partial f}{\partial x} = \begin{bmatrix} \frac{df_1}{dx_1} & \frac{df_1}{dx_2} & \frac{df_1}{dx_3} & \frac{df_1}{dx_4} \\ \frac{df_2}{dx_1} & \frac{df_2}{dx_2} & \frac{df_2}{dx_3} & \frac{df_2}{dx_4} \\ \frac{df_3}{dx_1} & \frac{df_3}{dx_2} & \frac{df_3}{dx_3} & \frac{df_3}{dx_4} \\ \frac{df_4}{dx_1} & \frac{df_4}{dx_2} & \frac{df_4}{dx_3} & \frac{df_4}{dx_4} \end{bmatrix} \quad (39)$$

This method can be generalized for any vector of any length. Using this method, we obtain the above defined matrices A_k and C_k by differentiating matrices $f(x_k, u_k)$ and $h(x_k)$ with respect to x_k as follows:

$$A_k = f'(\hat{x}_k, u_k) = \begin{bmatrix} 1 - \frac{R_s T_s}{L_s} & 0 & \frac{\lambda_f T_s \sin \hat{x}_k(4)}{L_s} & \frac{\lambda_f T_s \hat{x}_k(3) \cos \hat{x}_k(4)}{L_s} \\ 0 & 1 - \frac{R_s}{L_s} & \frac{-\lambda_f T_s \cos(\hat{x}_k)}{L_s} & \frac{\lambda_f T_s \hat{x}_k(3) \sin \hat{x}_k(4)}{L_s} \\ \frac{-3\lambda_f T_s \sin(\hat{x}_k(4))}{2J} & \frac{3\lambda_f T_s \cos(\hat{x}_k(4))}{2J} & 1 - \frac{BT_s}{J} & \frac{-3\lambda_f T_s [\hat{x}_k(1) \cos(\hat{x}_k(4)) + \hat{x}_k(2) \sin(\hat{x}_k(4))]}{2J} \\ 0 & 0 & T_s & 1 \end{bmatrix} \quad (40)$$

$$C_k = h'_k(\hat{x}_k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (41)$$

Now, by substituting A_k and Q into state and covariance propagation equations we update the state estimate and error covariance:

$$\tilde{x}_{k+1} = f_k(\hat{x}_k, u_k) \quad (42)$$

$$\tilde{P}_{k+1} = A_k P_k A_k^T + Q \quad (43)$$

Finally, we update the state estimate and error covariance using Kalman filter equations as follows:

$$K_k = \tilde{P}_k C_k^T (C_k \tilde{P}_k C_k^T + R)^{-1} \quad (44)$$

$$\hat{x}_{k+1} = \tilde{x}_{k+1} + K_k [y_k - h(\hat{x}_k)] \quad (45)$$

$$P_{k+1} = (I - K_k C_k) \tilde{P}_k \quad (46)$$

5.2. Synchronous Motor State Estimation Using Hybrid Kalman Filter

In this sub-section, all the equations are similar to extended Kalman filter (Sub-section 5-1) with the difference that system dynamics is considered to be continuous-time. In another words there is no need to discretize system dynamics. Furthermore, we solve state and covariance propagation equations (12) using rectangular integration (Euler integration). Note that when the sample time is small, we can use Euler integration. Otherwise, other integration methods (like trapezoidal integration, runge-kutta, etc.) must be used because this causes increasing the estimation error. It is notable that when the nonlinear system has more nonlinear terms, system sampling time must be smaller in order to simulate the complete behavior of the system.

6. FILTER TUNING

An important step in Kalman filter design is providing numerical evaluation of filter parameters by the initial conditions x_0 and covariance matrices P_0 , Q , and R . This process is called filter tuning that includes an iterative search for the effective values that may enhance the estimate efficiency.

The covariance value Q represents modeling inaccuracy, system perturbation, and known noise in voltage measurement. On the other hand, noise covariance R indicates measurement noise of current sensors.

Changing covariance matrices Q and R affects both transient and permanent performances of the filter. Increasing Q may indicate large noises stimulating the system or model uncertainty increase. This increases the values of the covariance elements. Filter gains will increase measurement weight which makes the filter transient performance faster. Similarly, increasing the covariance matrix R indicates that the measurements are exposed to strong destructive noise and must be less weighed by the filter. As a result, the gain matrix K will be decreased and the filter will be slower.

For the covariance matrix of the initial state P_0 , the main diagonal elements indicate the variance (or mean square error) in knowing the initial states. Changing P_0 generates different transient response but does not change neither transient time nor steady state conditions. In this paper, to perform the simulation, the filter is tuned as follows:

$$R = 0.001I(2); \quad P_0 = 0.1I(4); \quad (47)$$

$$x_0 = [0 \ 0 \ 0 \ 0]^T; \quad \hat{x}_0 = [-0.1 \ 0.2 \ 0.02 \ -0.01]^T; \quad (48)$$

Where $I(2)$ and $I(4)$ indicate the unit matrices with dimensions 2×2 and 4×4 respectively.

7. SIMULATION RESULTS

Although our measurements include winding currents, we can estimate motor's angular velocity and position using discrete-time extended Kalman filter and hybrid Kalman filter. Simulations are done using MATLAB and the results are shown in the following figures:

As shown in figures (2), (3), and (4), state variable are well estimated. Furthermore, drawing the angular velocity and position within their $\pm\sigma$ range in figures (3-b) and (4-b) indicates the good performance of this estimator.

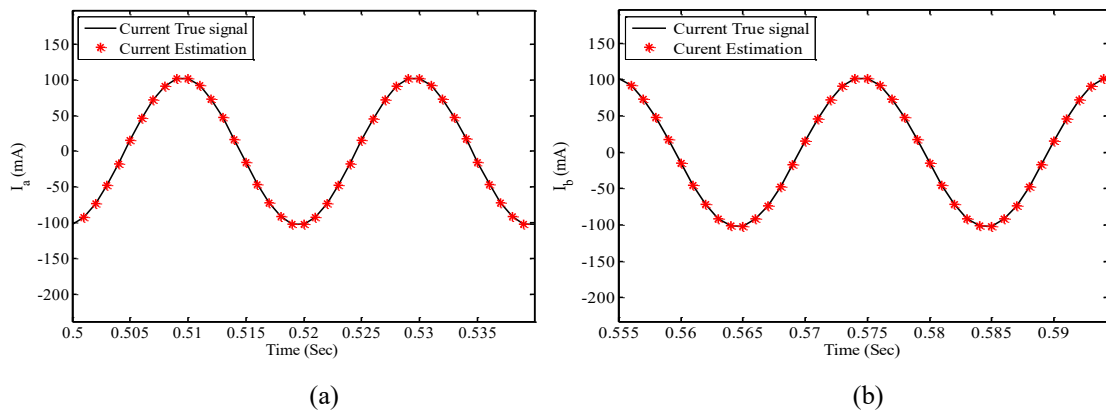


Fig. 2. Results of discrete-time extended Kalman filter estimator. (a) and (b) represent respectively A and B winding currents along with their estimate

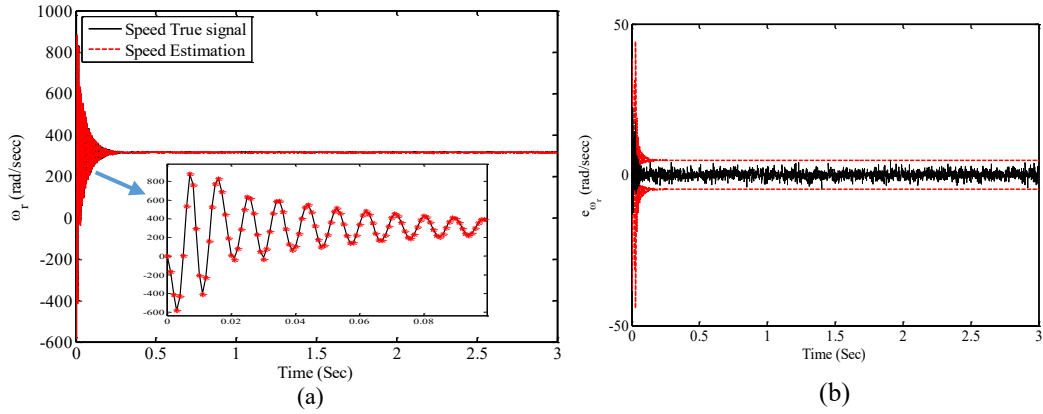


Fig. 3. Results of discrete-time extended Kalman filter estimator. (a) rotor's angular velocity and its estimate (b) angular velocity estimate error and its range.

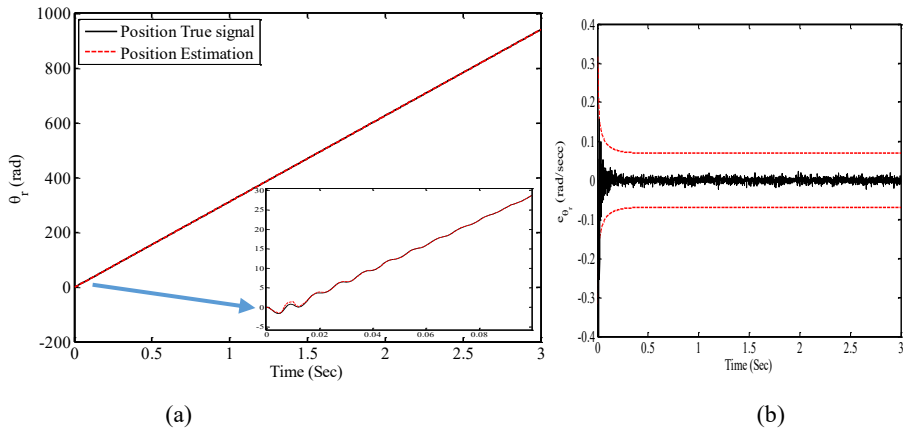


Fig. 4. Results of discrete-time extended Kalman filter estimator. (a) rotor's angular position and its estimate (b) angular position estimate error and its $\pm\sigma$ range.

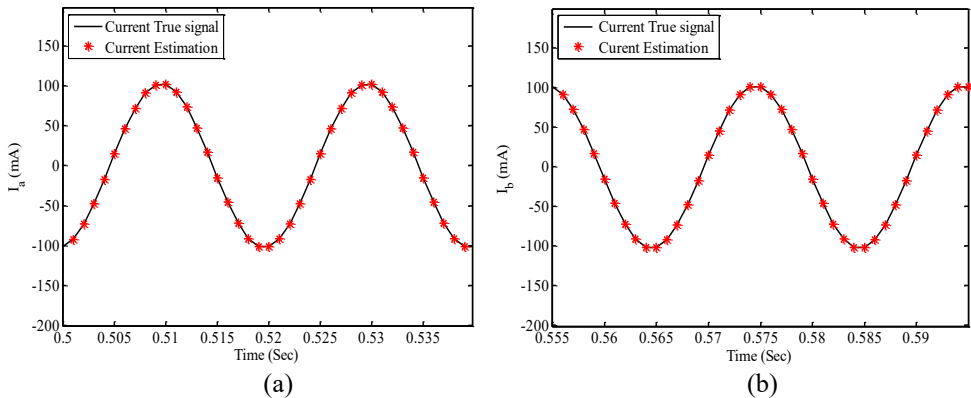


Fig. 5. Results of hybrid extended Kalman filter estimator. (a) and (b) represent respectively A and B winding currents along with their estimate.

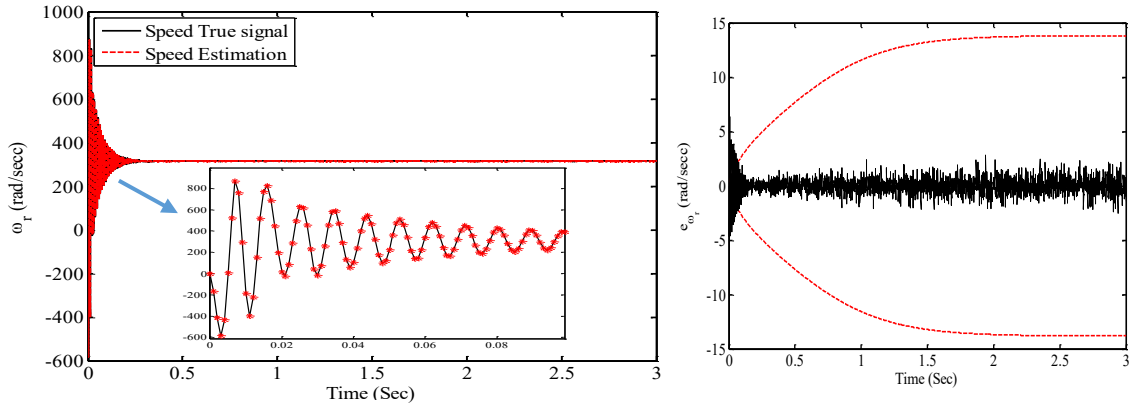


Fig. 6. Results of hybrid extended Kalman filter estimator. (a) rotor’s angular velocity and its estimate (b) angular velocity estimate error and its range.

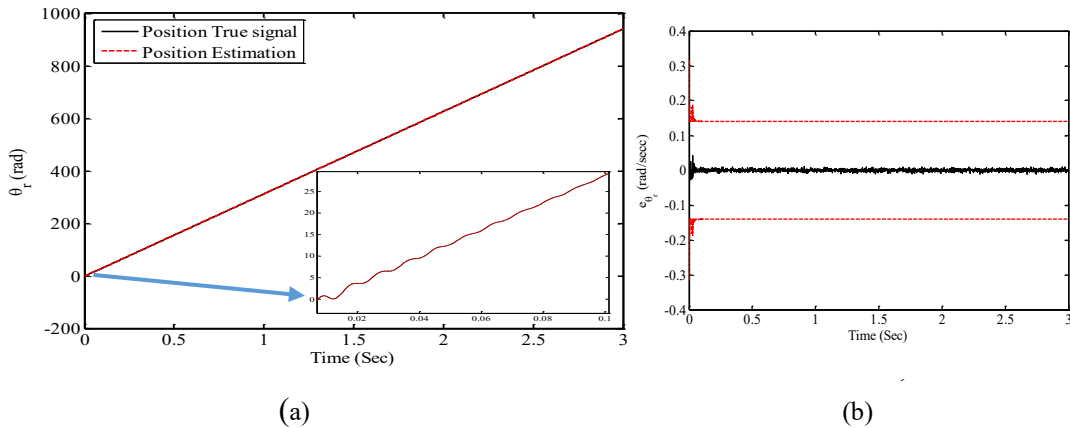


Fig. 7. Results of hybrid extended Kalman filter estimator. (a) rotor’s angular position and its estimate (b) angular position estimate error and its $\pm\sigma$ range.

As shown in figures (5), (6), and (7), state variable are well estimated. Furthermore, drawing the angular velocity and position within their $\pm\sigma$ range in figures (6-b) and (7-b) indicates the good performance of this estimator. To compare the two simulation methods, table (1) contains the root mean square error (RMSE) of the two estimators.

Table 1. Comparison of RMSE of discrete-time extended Kalman filter and hybrid extended Kalman filter for permanent magnet synchronous motor

State variable	RMSE_DEKF	RMSE_HEKF
I_α	0.0512	0.0203
I_β	0.0485	0.0197
ω_r	1.3776	0.7414
θ_r	0.0334	0.0281

As shown in the above table, the values of the root mean squares error of the synchronous motor states resulted from hybrid extended Kalman filter estimator are less than that of discrete-time extended Kalman filter estimator. This comparison indicates that the performance of hybrid extended Kalman filter estimator is better than that of discrete-time extended Kalman filter estimator.

8. CONCLUSION

In this paper, to estimate the permanent magnet synchronous motor angular velocity and position, discrete-time extended Kalman filter and hybrid extended Kalman filter are designed and simulated. The angular velocity and position are estimated using the measurement of winding current. Simulation results show that an acceptable estimate to the states of a nonlinear system can be obtained using discrete-time extended Kalman filter and hybrid extended Kalman filter. It is shown also that for a continuous-time nonlinear system with discrete-time measurements, the root mean squares error for hybrid extended Kalman filter estimator is much less than that of discrete-time extended Kalman filter. This indicates a better performance of hybrid extended Kalman filter for such systems.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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