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Design Of a New Optimal Controller for a Particular Class of Chaotic Systems Using the Artificial Bee Colony Algorithm (ABC)

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ABSTRACT

The primary aim of this paper is to devise an optimal regulator for stabilizing a distinct class of chaotic systems through a systematic two-step approach. Initially, the chaotic system undergoes transformation into state-dependent equations. Subsequently, the State Dependent Riccati Equation (SDRE) is tackled via the power series method, facilitating the determination of the optimal control law. Ensuring a suitable regulatory response involves the utilization of an intuitive optimization algorithm of a naturalistic nature, with a focus on optimizing the weight matrices within the SDRE equation. Employing the Artificial Bee Colony (ABC) algorithm, we derive the weighted matrices, leveraging the honey bee algorithm to fine-tune the gain coefficients by minimizing the chosen fitness function. The fitness function, represented as the sum of squares of system state errors, proves instrumental in achieving effective stabilization of the chaotic system, minimizing error, enhancing response speed, and reducing control costs. Through simulation, we scrutinize the effectiveness of regulators designed to stabilize and control chaotic systems, particularly comparing the regulatory performance of this algorithm against the SDRE method.

1. INTRODUCTION

Chaos theory, a mathematical discipline, delves into the intricacies of dynamical systems highly responsive to initial conditions. Despite their deterministic nature, these systems lack predictability [1-2]. Demonstrating heightened sensitivity to initial conditions, chaotic systems undergo substantial changes even with minimal variations [3]. Over more than two decades, in-depth exploration of dynamical systems has unveiled the widespread prevalence of chaos in natural and engineering domains. The applications of chaos theory span across diverse fields, encompassing cryptography [4-5], robotics [6-7], biology [8], quantum physics [9], electrical engineering [10], and beyond.

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Efforts to synchronize and control chaotic systems have yielded numerous methodologies. Leandro and Diego achieved synchronization in a nonlinear chaotic discrete system using a PID controller. The PID controller gain coefficients were determined through Tribe's and modified Tribe's algorithms [11]. Çimen synchronized an energy resource system using two distinct methods—active control and adaptive control—for these controllers [12]. Amidst various techniques proposed for controlling chaotic systems, optimal control methods stand out for their enhanced capabilities [13-17]. Beeler investigated optimal controllers for a class of affine nonlinear systems using power series, while Feng delved into robust optimal control for nonlinear systems, including chaotic ones [16-17].

In the realm of optimization, various nature-inspired algorithms, such as Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and the ABC algorithm, have emerged. The artificial bee colony algorithm, inspired by natural bee behavior, was pioneered by Kara Boga [18]. Wan et al. conducted a comparative analysis, evaluating the performance of ABC against GA and PSO evolutionary algorithms [19]. The ABC algorithm's simplicity and its ease of parameter adjustment have led to its utilization in diverse research domains, spanning function optimization [20-21], cluster analysis, image processing, vehicle routing, and more [22-24].

This article focuses on solving the State Dependent Riccati Equation (SDRE) using the power series, leading to the derivation of an optimal control law. The ABC algorithm is employed to determine optimal values for the weighting matrices Q and R in the SDRE equation. The structure of the paper is as follows: Section 1 introduces the topic. Section 2 outlines the process of designing optimal control using power series based on the SDRE method. Section 3 provides a comprehensive overview of the Artificial Bee Colony (ABC) algorithm in determining the optimal weight matrices Q and R. Section 4 presents the simulation results. Finally, Section 5 concludes the paper.

2. VIEW OF OPTIMAL CONTROL USING POWER SERIES

The considered nonlinear chaotic system that has nonlinearity in state and control as following form:

$$\dot{x} = f(x) + g(x,u) \tag{1}$$

The system equation (Eq. (1)) can be rewrite as following form:

$f(x) = A(x)$ and $g(x,u) = B(x,u)u$

where the matrices $A(x)$ and $B(x,u)$ are separate into constant part and a variable part, separately as follows:

$$A(x) = A_0 + \Delta A(x) \text{ and } B(x,u) = B_0 + \Delta B(x,u) \tag{2}$$

The goal is to find a control vector that minimizes the following cost function:

$$J(x_0,u) = \frac{1}{2}\int_0^{\infty} (x(t)^T Q x(t) + u(t)^T R u(t))dt \tag{3}$$

In order to finding optimal control of chaotic systems using power series algorithm, the above cost function (Eq. (4)) must be solved.

$$\begin{aligned} A(x)^T P(x,u) + P(x,u)A(x) \\ - P(x,u)B(x)R^{-1}B(x,u)^T P(x,u) \\ + Q = 0 \end{aligned} \tag{4}$$

In this approach, $P(x(t),u(t))$ can be considered as follows Eq. (5):

$$P(x(t),u(t)) = L_0(x(t),u(t)) + \varepsilon L_1(x(t),u(t)) + \varepsilon^2 L_2(x(t),u(t)) + \cdots$$

$$= \sum_{j=0}^{\infty} \varepsilon^j L_j(x(t), u(t)) \tag{5}$$

In the above relations, j[th] term of power series is $L_j$and also$\varepsilon$is a positive value. In general, the equations are written as Eq. (5):

$$L_j(A_0 - B_0 R^{-1} {B_0}^T L_0) + ({A_0}^T - {L_0}^T B_0 R^{-1} {B_0}^T) L_j + L_{j-1} \Delta A + \Delta A^T L_{j-1} - \sum_{i=1}^{j-1} (L_i\, B_0 R^{-1} {B_0}^T L_{j-i}) - \sum_{i=0}^{j-1} L_i\, (B_0 R^{-1} \Delta B^T + \Delta B R^{-1} {B_0}^T) L_{j-1-i} - \sum_{i=0}^{j-2} L_i\, \Delta B R^{-1} \Delta B^T L_{j-2-i} = 0 \tag{6}$$

With obtaining$P(x, u)$, the optimal control law $u(x(t)$ is rewritten as follows:

$$u_{(n+1)}(x(t)) = -R^{-1} B(x(t), u_{(n)}(t))^T \sum_{j=0}^{k_P} L_j(x(t), u_{(n)}(t)) x\,(t) \tag{7}$$

Where k_P is the number of sentences in matrix L_j.

3. A GENERAL VIEW OF ABC ALGORITHM

The Artificial Bee Colony (ABC) algorithm has been widely used in various research studies due to its simplicity and the need for fewer control parameters[25]. Liao (2013) further supports this, showing that the performance of ABC is comparable to other population-based algorithms, and can be improved with the addition of local search [26]. Bolaji (2013) provides a comprehensive overview of the applications of ABC, highlighting its successful implementation in discrete and continuous optimization problems [27]. Ozkis (2013) presents the accelerated ABC (A-ABC) algorithm, which enhances the local search ability and convergence speed of the standard ABC algorithm [28]. These studies collectively demonstrate the versatility and effectiveness of the ABC algorithm in various optimization and artificial intelligence applications.

In this paper, the ABC algorithm is used to optimize weight matrices. Selection processes in the artificial bee colony algorithm include the following steps:

3.1. Global selection process

At first, sources of food (initial questions of the problem) are randomly assigned through equation (1).

$$x_{ij} = x_j^{{min_j}^{{max_j}^{min}}} \tag{8}$$

where i=1,2,…,SN (SN is the number of food sources);j=1,2,…,D (D is the number of parameters);$x_j^{min}$is min and $x_j^{max}$is maximum values of parameter j.

3.1.1. The process of local selection by the Employed Bees

Moved Employed Bees to the food sources, which is the location of the bees in the space of the problem. Actually, Each Employed Bees selects a neighborhood with randomly and moves to it by equation (9). Where j and k are randomly chosen parameters and neighborhoods, respectively and $\phi_{ij}$is a random number within [-1, 1]. Also,$V_i$is the new target food source.

$$V_{ij} = x_{ij} + \phi_{ij}(x_{ij} - x_{kj}) \tag{9}$$

If the new position (new food location) provided a greater amount of nectar, the bee would remain in the new area. Otherwise, it will return to its previous location. In fact, the number of consecutive times the bee travels without improvement is an indicator. If this indicator reaches a certain level, it indicates a lack of nectar in the area and the bee must leave the food location.

3.2. Local selection process by onlooker bees

Move onlooker bees probabilistically to food sources depending on the roulette wheel using the following equations and the determination of new neighborhoods as in step2:

$$p_i = \frac{fitness_i}{\sum_{j=1}^{SN} fitness_i} \tag{10}$$

$$fitness_i = \begin{cases} \dfrac{1}{1 + fit_i}, fit_i \geq 0 \\ 1 + abs(fit_i), fit_i \prec 0 \end{cases} \tag{11}$$

Where fit_i is the value of objective function of the food source x_i and $fitness_i$ the nectar amount (fitness value) of the food sourcex_i.

3.3. Random selection process by Scout Bees

If an unsuitable nectar source is detected, the Scout bees vacate the area and search for other options at random. If the trial index is reached and a better source is not found, the Scout bees will establish a new food source by chance using Eq. (8).

4. SIMULATION RESULTS

The equations describing the chaotic system of three degrees of freedom are as follows [29]:

$$\begin{aligned} \dot{x} &= a(y - x) \\ \dot{y} &= -4ay + xz + mz^3 \\ \dot{z} &= -adz + x^3y + bz^2 \end{aligned} \tag{12}$$

In Which A, B, D And M Are Constant Parameters of The System. When The Behavior of The System Appears, The System Parameters Are A = 1.8, B = -0.07, D = 1.5and M = 0.12. The Electronic Circuit Schematic of The Chaotic System Is Seen In (Fig. (1)). The State Variables of Chaotic System and Three-Dimensional Phase Diagram Are Shown Respectively In (Figs. (2) To (6)):

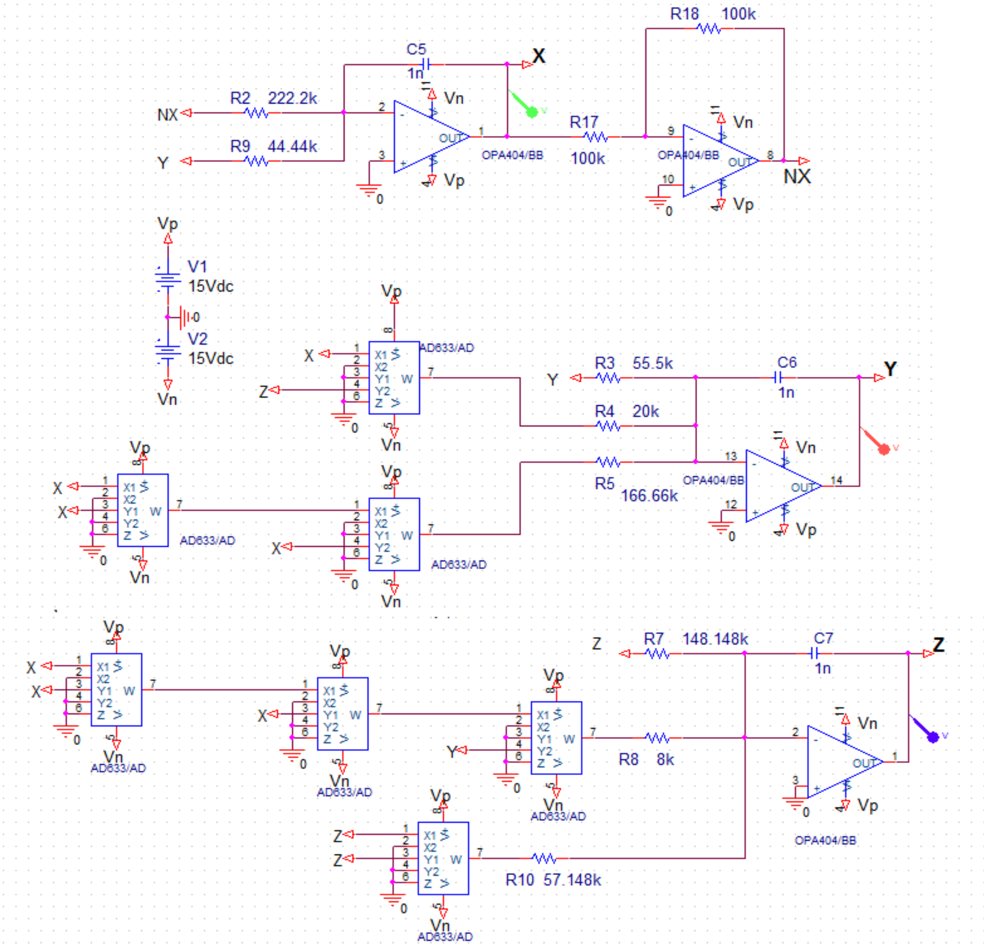

Fig.1. Electronic circuit schematic of the chaotic system

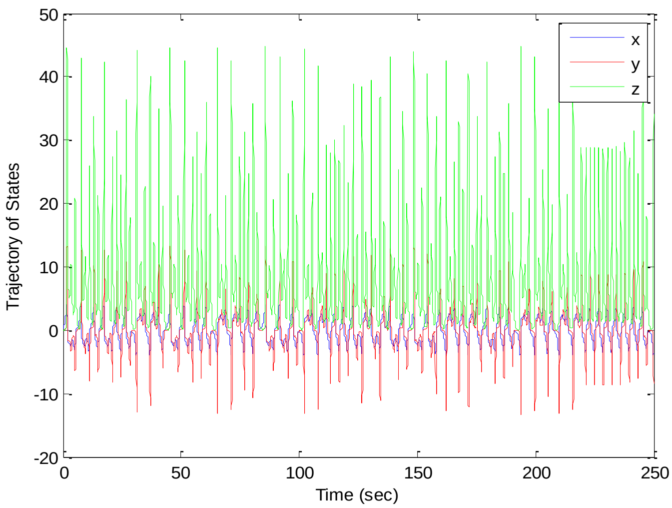

Fig. 2. Trajectory of states in chaotic system.

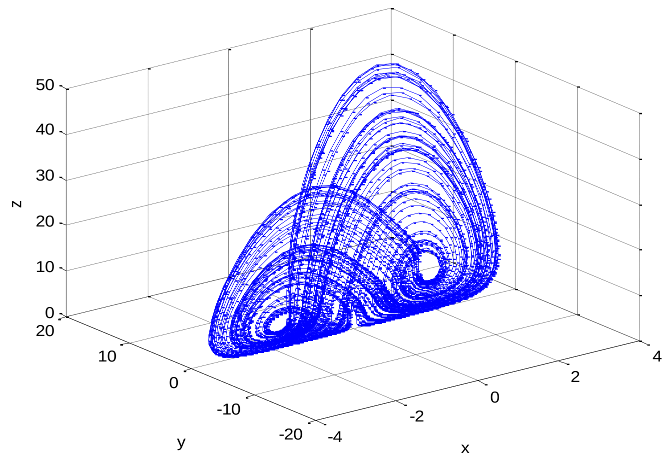

Fig. 3. Three-dimensional phase diagram of chaotic system.

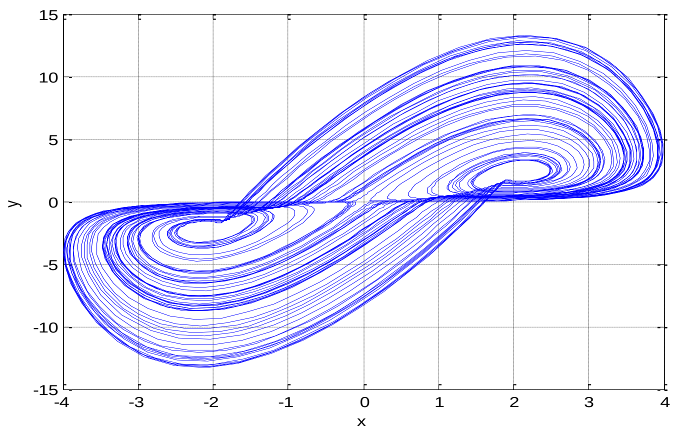

Fig. 4. phase portraits of state variables xandy.

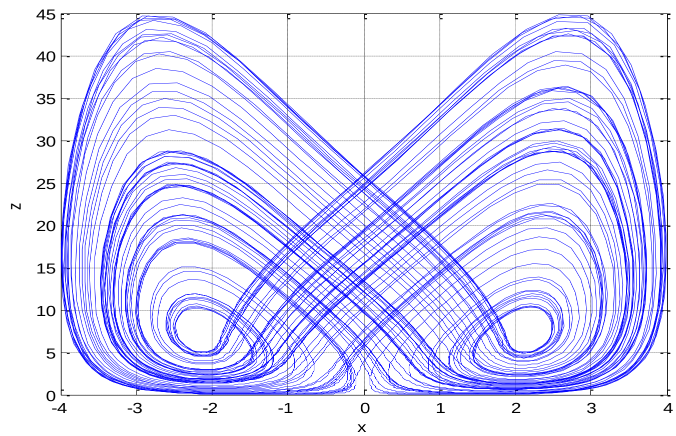

Fig. 5. phase portraits of state variables xandz.

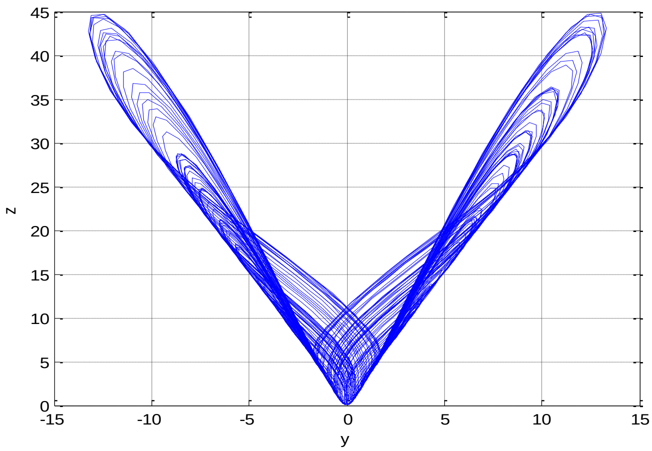

Fig. 6. phase portraits of state variables xandz.

4.1. Designing an optimal regulator for chaotic systems using the SDRE method

Description equations are obtained by adding two the control signals u_1 and u_2 into the chaotic system as follows:

$$\begin{cases}\dot{x} = a(y-x) \\ \dot{y} = -4ay + xz + mx^3 + (1+u_1)u_1 \\ \dot{z} = -adz + x^3y + bz^2 + u_2 \\ x(0) = [x_0 \quad y_0 \quad z_0]^T = [0.25 \quad -0.25 \quad 0.25].\end{cases} \tag{13}$$

Global form in the power series algorithm is shown as follows:

$$\dot{x} = A(x)x + B(x,u)u \tag{14}$$

By separating and replacing the constant parameters of chaotic system, the following equation is obtained:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \left(\begin{bmatrix}1.8 & -1.8 & 0\\ 0 & -7.2 & 0\\ 0 & 0 & -2.7\end{bmatrix} + \begin{bmatrix}o & o & 0\\ 0.12x^2 & 0 & x\\ 0 & x^3 & -0.07z\end{bmatrix}\right)\begin{bmatrix}x\\ y\\ z\end{bmatrix} + \left(\begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix} + \begin{bmatrix}0 & 0\\ u_1 & 0\\ 0 & 0\end{bmatrix}\right)\begin{bmatrix}u_1\\ u_2\end{bmatrix} \tag{15}$$

Weighting matrices Q and R in the power series method are chosen as follows:

$$Q = I_{3\times 3} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \text{and} R = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \tag{16}$$

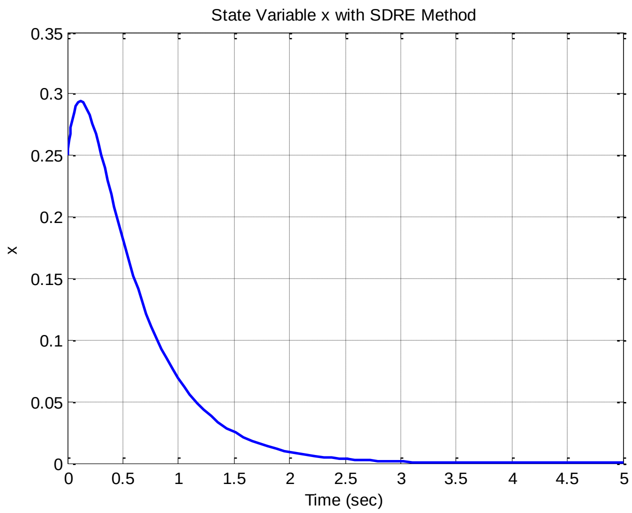

Fig. 2. Time response of state variable x with SDRE method

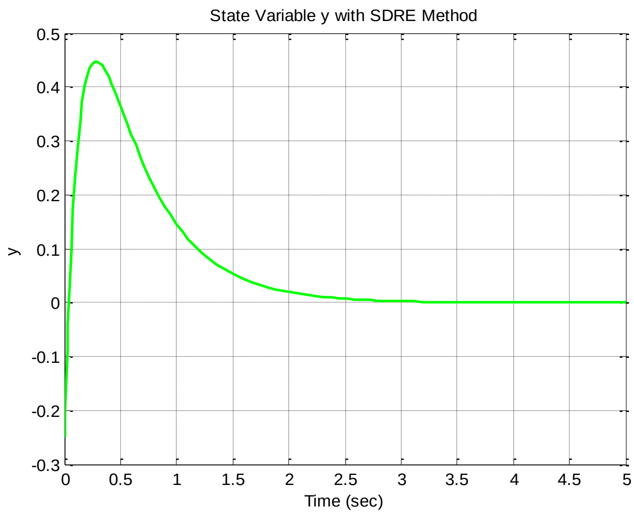

Fig. 3. Time response of state variable y with SDRE method

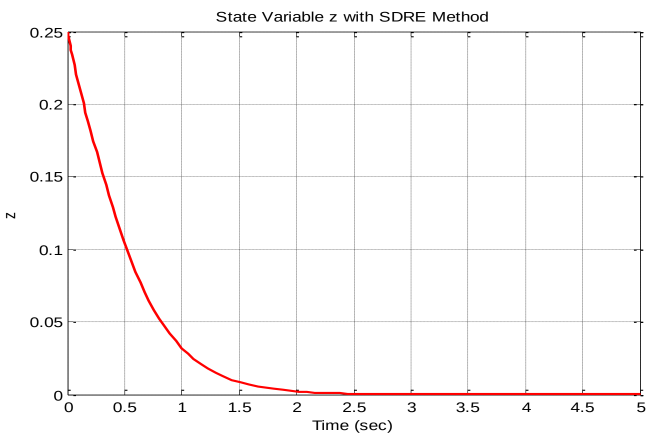

Fig. 4. Time response of state variable z with SDRE method

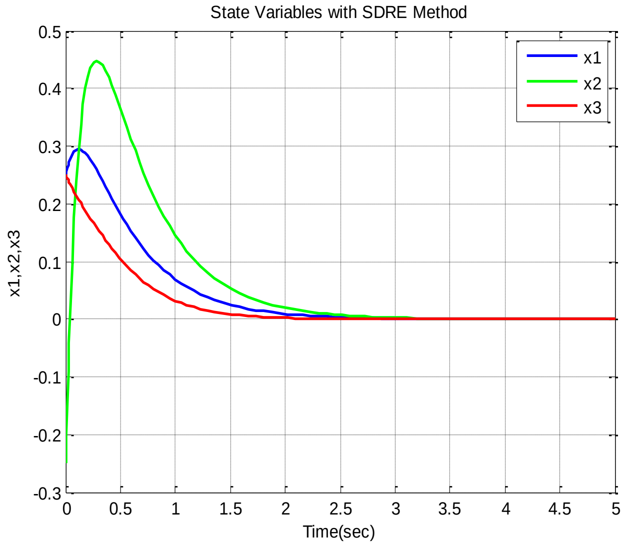

Fig. 5. Time response of state variables x, y and z with SDRE method

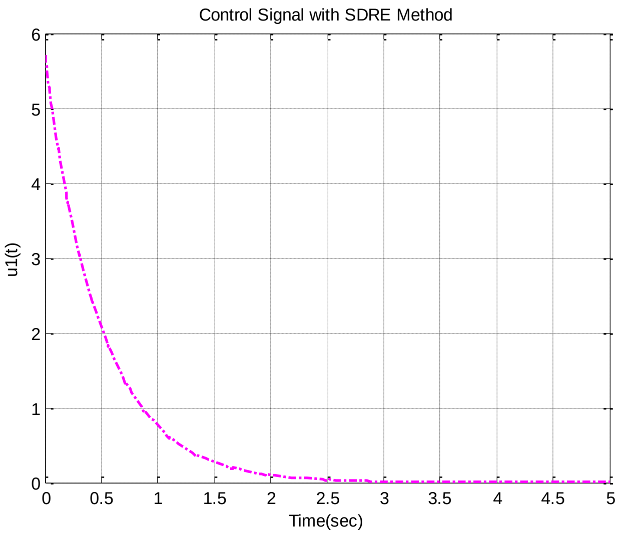

Fig. 6. Time response of the control signal U1 with SDRE method

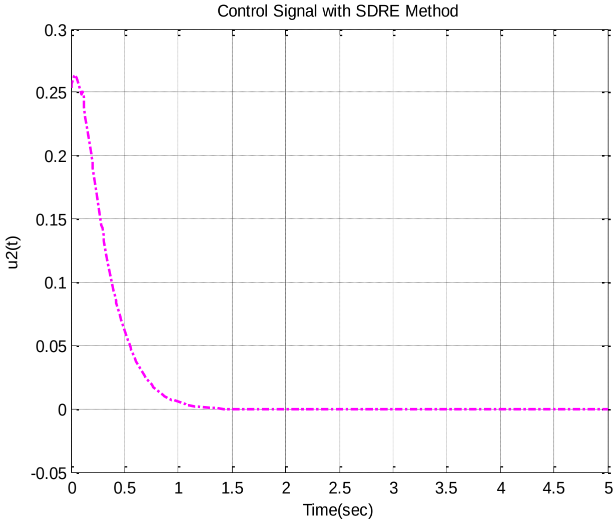

Fig. 7. Time response of the control signal U2 with SDRE method

4.1.1. Designing an optimal regulator for chaotic systems using the ABC algorithm

To design the optimal regulators using the ABC algorithm, the criteria of error reduction has been used as follows in the Eq. (17). And also, the target parameters to be optimized are k_1,k_2, k_3, $k_4$and k_5:

$$Q = \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix} \text{ and} R = \begin{bmatrix} k_4 & 0 \\ 0 & k_5 \end{bmatrix} \quad (17)$$

The design parameters in the ABC algorithm used to optimize the weight matrices are listed in the following table 1:

Table 1. Control Parameters of ABC algorithm:

the number of colony size (employed bees+ onlooker bees)	**NP=20**
the number of food sources equals the half of the colony size	Food Number=NP/2
a food source which could not be improved through "limit" trials is abandoned by its employed bee	limit=100
the number of cycles for foraging {a stopping criteria}	Max Cycle=10

By applying this algorithm, weighted matrices are obtained as follows:

$$Q = \begin{bmatrix} 6.4708 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 10 \end{bmatrix} \text{ and } R = \begin{bmatrix} 2.3774 & 0 \\ 0 & 8.3307 \end{bmatrix} \quad (18)$$

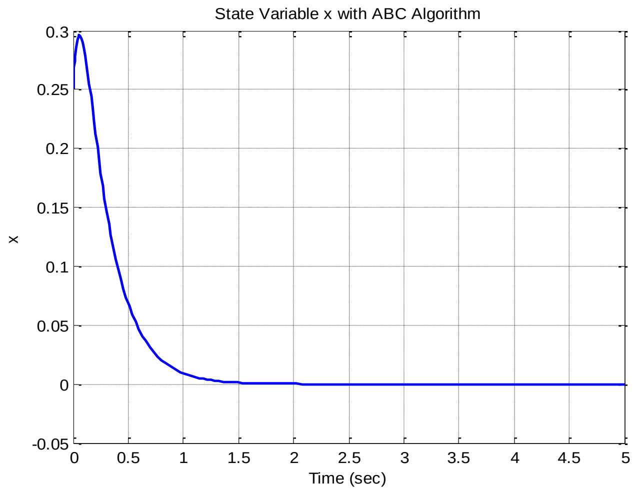

Fig. 8. Time Response of State Variable x With ABC Algorithm

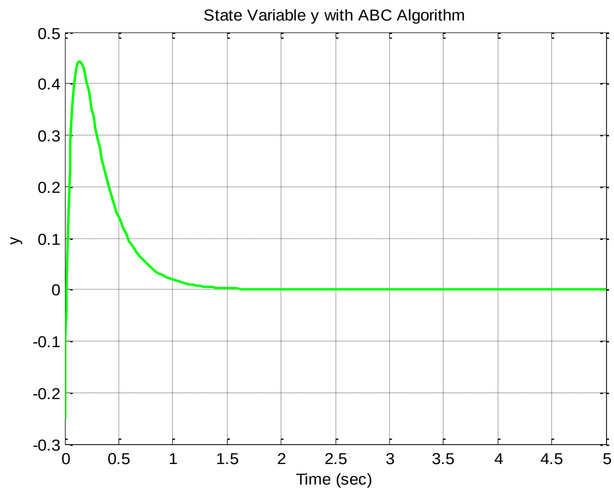

Fig. 9. Time Response of State Variable y With ABC Algorithm

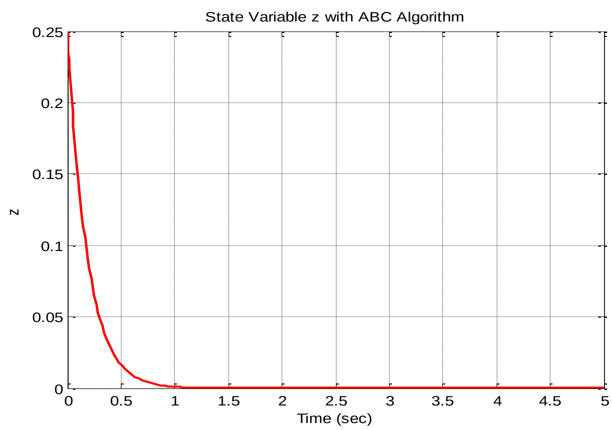

Fig. 10. Time Response of State Variable z With ABC Algorithm

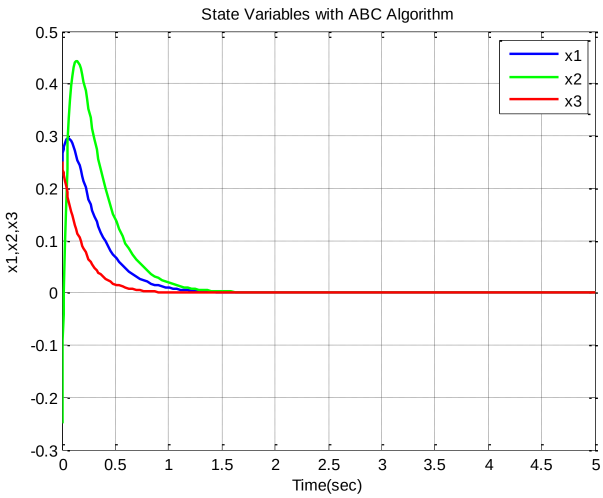

Fig.11. Time Response of State Variable x, y and z With ABC Algorithm

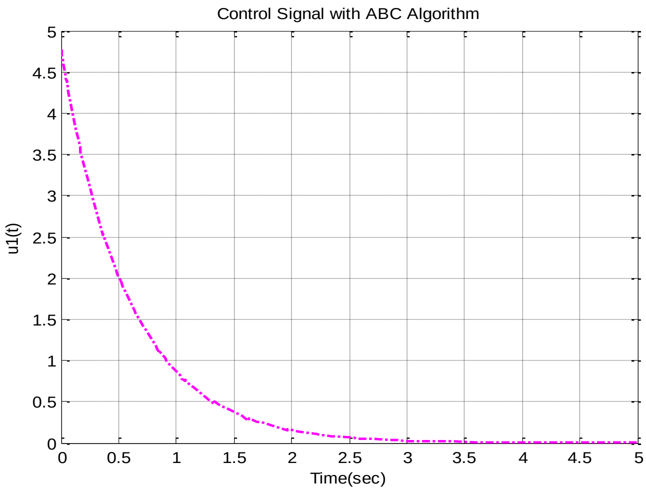

Fig. 12. Time response of the control signal U1 with ABC algorithm

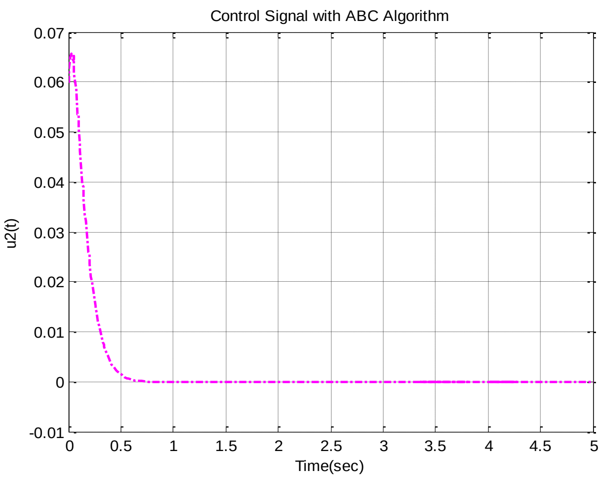

Fig. 13. Time response of the control signal U2 with ABC algorithm

5. CONCLUSION

This paper presents a novel technique for developing the optimal regulator for a specific category of chaotic systems via the honey bee algorithm, a nature-inspired optimization algorithm. The approach involves a two-stage process. Firstly, an optimal regulator is formulated using power series based on the state of state-dependent reticulate equations. The key advantage of this regulatory function is its applicability in developing regulators for chaotic systems that have been linearized in both state and control. In the second phase, optimized weighting matrices are acquired through the use of the ABC algorithm, with the aim of enhancing system responsiveness. The main function of the optimal regulator designed with this algorithm is that:

1. It can be used to design the regulator of chaotic systems that have linearization in state and control.
2. By minimizing the fitness function, the values are more appropriate for the coefficients of the regulator's gain. So that a new optimal design regulation is able to control the chaotic system with faster regulator response and narrower control (smaller) control laws.

In simulations have been shown, the abilities of proposed regulator.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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