ISSN Online: 2821-1693

Transactions on Machine Intelligence

Journal Homepage: https://tmachineintelligence.ir/index.php/TMI

Design and Implementation of a Discrete-Time Unscented Kalman Filter (DTUKF) Based on Genetic Algorithm to Enhance the Performance of Nonlinear Navigation Systems

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ARTICLE INFO

Article History:
Received 10 February 2020
Received in revised form 14 April 2020
Accepted 5 June 2020
Available online 5 June 2020

Keywords:
Genetic Algorithm, Nonlinear Inertial Navigation System, Extended Kalman Filter, Linear Kalman Filter, Unscented Kalman Filter

ABSTRACT

Nonlinear inertial navigation systems are considered among the most important navigation systems, featuring advantages such as independence from external communication, real-time speed and position calculation, and suitable bandwidth, making them very popular in vehicle navigation. These systems consist of an inertial measurement unit comprising three orthogonal accelerometers and three gyroscopes used to determine the position, speed, and direction of vehicle movement, calculating the vehicle's position with minimal error over short distances. However, over time, due to errors in the gyroscopes and accelerometers and consecutive integrations of their outputs, the estimated position error increases. To achieve higher accuracy, especially in long-term navigation, a global positioning system (GPS) is used for its complementary properties with the inertial navigation system (INS). This paper discusses and analyzes the application of two Kalman filters linear Kalman filter and unscented Kalman filter on nonlinear navigation systems. A discrete-time unscented Kalman filter (DTUKF) is utilized to estimate the position, speed, and state of a nonlinear system, which in this case is an integrated navigation system. By accurately selecting a set of sigma points from a Gaussian distribution and propagating these points through the nonlinear function, estimation is performed with significantly higher accuracy. The unscented transformation allows for the selection of the distribution of these points and control of higher-order error using design parameters. Comparing this filter with the linear Kalman filter reveals the superior performance of the unscented Kalman filter due to not linearizing the nonlinear system, thereby reducing system error. Notably, this paper is the first to use a genetic algorithm to optimize the Q and R noise matrices to achieve minimal variance and optimal mean convergence to reference values.

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http://dx.doi.org/10.47176/TMI.2020.64

1. INTRODUCTION

The emergence of statistical signal processing methods dates back to the early 19th century [1-9]. Significant advancements in estimation theory and optimal filters are attributed to the work of Wiener and Kalman in the mid-20th century. An optimal estimator is a computational algorithm that, by processing measured data and considering some statistical characteristics, provides a good estimate of the desired variables based on an optimality criterion. Hence, the Kalman filter family is used for combining information and estimating states. The Kalman filter is essentially an optimal state estimation process applied to a dynamic system subject to random disturbances [10]. More precisely, the Kalman filter is a recursive, unbiased algorithm that minimizes error variance, optimally estimating the unknown state of a dynamic system using noisy measurements. The first filter examined by Kalman assumed a linear system. Therefore, the estimation error is high since real systems have a nonlinear nature [11]. In [12], the extended Kalman filter (EKF) is used for this purpose. Reference [13] employs a second-order extended Kalman filter to enhance the filter's accuracy in dealing with system nonlinearity. In [14], a nonlinear estimator filter is introduced using the $H\infty$ filter Riccati equation, which considers navigation system nonlinear errors. The filter equation includes a degree of freedom to improve filter stability and performance. In some nonlinear systems, the complexity of the problem depends on sensor fusion methods, typically using the Kalman filter algorithm, especially the extended Kalman filter (EKF) [15,16]. It is suggested to use nonlinear Kalman filters instead of linear Kalman filters (LKF) to increase estimation accuracy [15].

The most common algorithm used for multi-sensor fusion is the extended Kalman filter, which operates based on system dynamics linearization, leading to suboptimal application in the standard Kalman filter's recursive estimation [17]. The extended Kalman filter works well in practice and is theoretically significant for minimizing mean squared error (MSE) variance. A major drawback of this filter is its heavy reliance on a predefined dynamic model and the use of the Jacobian matrix in linearizing nonlinear systems, which reduces parameter estimation performance in nonlinear systems. To improve estimation and filtering results, the designer must have comprehensive prior information about both the dynamic process and the measurement model. Additionally, it is assumed that both the process and measurements are corrupted by zero-mean white Gaussian noise. If the input data poorly represent the actual model, the obtained estimates will be unreliable. In designing some nonlinear systems, uncertainties in the system model cannot be represented by a linear state-space model, as the linear model includes modeling errors [18,19].

In the extended Kalman filter, due to the linearization process, issues such as reduced filter performance and divergence arise. To better address the nonlinearity of real systems, this paper proposes using a discrete-time unscented Kalman filter (DTUKF) [20]. The discrete-time unscented Kalman filter was first introduced by Julier and Uhlmann in 1997 [20,21]. This filter consists of two parts: prediction and update. For linear systems, the linear Kalman filter is used for state estimation. For nonlinear systems, the EKF can be used for state estimation, but as mentioned earlier, this filter has estimation errors due to system linearization. The unscented Kalman filter is superior to the extended version not only theoretically but also in many practical implementations. The DTUKF was first proposed by Julier for nonlinear state estimation in control theory [22]. This filter is an approximation method for nonlinear distribution that uses a limited number of accurately selected sigma points to propagate the state distribution. Unlike the extended Kalman filter, which uses the Jacobian matrix in the linearization process and achieves first-order accuracy, the unscented Kalman filter uses a deterministic sampling approach to calculate a minimal set of sigma points and achieves at least second-order accuracy from the mean and covariance, significantly improving the performance of the integrated navigation system [23,24].

To further improve the performance of nonlinear systems, a genetic algorithm is used to determine the elements of the system noise covariance matrix Q and the measurement noise covariance matrix R. The genetic algorithm, recognized as a stochastic optimization method, was invented by John Holland in 1967. This algorithm is a search method in computer science for finding the optimal solution to the given problem. Genetic algorithms are a type of evolutionary algorithm that use principles of biology such as inheritance and mutation. Therefore, the minimal variance can be achieved with reference values, and the mean response over the desired time can converge optimally to the desired reference value [25].

The structure of this paper is as follows: After the introduction, the next section introduces the linear Kalman filter and its components. Then, in the third section, the discrete-time unscented Kalman filter is discussed. The fourth section analyzes the use of the genetic algorithm to improve the filter's performance. In the fifth section, a comparison of the two filters is made through simulation on a real nonlinear integrated navigation system. Finally, in the sixth section, the conclusions drawn from the performance of the linear Kalman filter and the discrete-time unscented Kalman filter are presented.

2. LINEAR KALMAN FILTER

As stated in the first section, the linear Kalman filter is more suitable for linear systems, providing optimal performance, but it does not perform as well in nonlinear systems due to linearization and neglecting some parameters. In this section, the components of the linear Kalman filter are introduced. Figure 1 shows the main components of the Kalman filter, which include the state vector and its covariance, the system model, the output vector (measurements) and its covariance, the output model, and the algorithm.

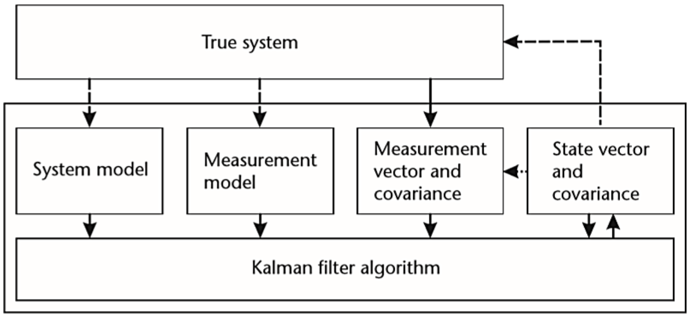


Fig.1. Components of the Kalman Filter

The state vector is essentially a set of parameters that describe the system, also referred to as the system states, which the Kalman filter aims to estimate. Each state can be either constant or time-varying. In most navigation applications, the states are the components of position or position error. In some cases, the velocity, position, and errors of navigation sensors can also be estimated as states. Associated with the state vector, an error covariance matrix is defined. This matrix indicates the uncertainties in the state estimates of the Kalman filter and the correlation between these estimation errors. The Kalman filter is a recursive process; therefore, the initial values of the state vector and the covariance matrix must be provided by the user to the filter.

The system model, also known as the process model or time propagation model, represents how the state vector and the error covariance matrix of the Kalman filter change over time. The uncertainty related to the system states must also increase over time. This increase is due to the fact that, in the absence of new measurements, the state estimates lose their validity due to unknown changes in the system. These changes can be caused by unmodeled dynamics or even sensor measurement noise. For instance, when the acceleration of an object is unknown, the uncertainty of its velocity will increase over time. These changes in the actual state values are referred to as system noise, and their statistical properties should be identified by the Kalman filter designer beforehand.

The output vector or measurement vector includes a set of simultaneous measurements of the system characteristics, which are also a function of the system states. Accompanying each output vector, an output noise covariance matrix is defined, which specifies the statistical properties of the measurement noise. The output model specifies the relationship between the output vector and the actual state vector of the system in the absence of measurement noise. In this filter, there are two types of uncertainties: one within the system and one in the output (measurement), known as system noise and output noise, respectively. These two uncertainties are introduced into the system through the uncertainty matrices, which are denoted as Q and R. The ratio of these two matrices indicates whether the Kalman filter should trust the output more or the system itself. The continuous-time system equations can be represented as follows:

$$\dot{\boldsymbol{x}} = F\boldsymbol{x} + G\boldsymbol{u} \tag{1}$$

In the above relation F is the system dynamics matrix, x is the state vector, G is the design matrix, and u is the system input vector, defined as follows:

$$F = \begin{bmatrix} F_{rr} & F_{rV} & 0 \\ F_{Vr} & F_{VV} & (\boldsymbol{f}^n \times) \\ F_{er} & F_{eV} & -(\boldsymbol{\omega}_{in}^n \times) \end{bmatrix} \qquad \boldsymbol{x} = \begin{bmatrix} \delta\boldsymbol{r}^n \\ \delta\boldsymbol{V}^n \\ \boldsymbol{\varepsilon}^n \end{bmatrix}$$
$$G = \begin{bmatrix} 0 & 0 \\ \boldsymbol{C}_b^n & 0 \\ 0 & -\boldsymbol{C}_b^n \end{bmatrix} \qquad \boldsymbol{u} = \begin{bmatrix} \delta\boldsymbol{f}^b \\ \delta\boldsymbol{\omega}_{ib}^b \end{bmatrix} \tag{2}$$

The elements of the vector u are considered as white noise, with its covariance matrix defined as follows:

$$E[\boldsymbol{u}(t)\boldsymbol{u}(\boldsymbol{T})^T] = Q(t)\delta(t-T) \tag{3}$$

In equation (3), δ represents the Dirac delta function. The matrix Q is defined as:

$$Q = diag\left(\sigma_{ax}^2\ \sigma_{ay}^2\ \sigma_{az}^2\ \sigma_{wx}^2\ \sigma_{wy}^2\ \sigma_{wz}^2\right) \tag{4}$$

In equation (4), δ_w and δ_a denote the standard deviations associated with the gyroscope and accelerometer, respectively. The implementation of the Kalman filter can be divided into two sections:

1. Data Correction
2. Data Prediction

First, the Kalman filter gain is computed, and then the error covariance and state vector are calculated to correct the previous estimates P_k^- and $\boldsymbol{x}_k^-$. This can be summarized as follows:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}$$
$$\boldsymbol{x}_k = \boldsymbol{x}_k^- + K_k(\boldsymbol{z}_k - H_k^T \boldsymbol{x}_k^-) \tag{5}$$
$$P_k = (I - K_k H_k) P_k^-$$

In the prediction phase, the state vector and error covariance are estimated as follows:

$$\boldsymbol{x}_{k+1}^- = \Phi_k \boldsymbol{x}_k$$
$$P_{k+1}^- = \Phi_k P_k \Phi_k^T + Q_k \tag{6}$$

3. DISCRETE-TIME NEUTRAL KALMAN FILTER

The most widely used algorithm for sensor fusion in real nonlinear systems is the Extended Kalman Filter (EKF). This filter operates based on the linearization of the system's dynamics, which leads to suboptimal application of the recursive estimation of the standard Kalman Filter [26,27]. Additionally, the EKF operates under the assumption of Gaussian measurement/process noise, which is not always valid. This assumption can significantly affect the performance of nonlinear systems and may even cause the algorithm to diverge. Consequently, the performance of the control loop that utilizes the system's state vector estimation based on the EKF may not be satisfactory. To overcome the limitations of the EKF, two different approaches for estimating the states of nonlinear dynamic systems have been proposed:

1. Sigma Point Kalman Filters (SPKF), specifically the Discrete-Time Unscented Kalman Filter (DTUKF).
2. Particle Filters.

It has been proven that the Unscented Kalman Filter (UKF) performs significantly better than the EKF in many practical applications. Implementing a Sigma Point Kalman Filter is considerably easier and does not require the extraction of Jacobian matrices or computations as needed for the EKF [28,29]. The state distribution in these filters is approximated using a Gaussian random variable, represented by a minimal set of appropriately weighted sample points.

A critical issue in nonlinear systems is that transforming a probability density function (PDF) through a nonlinear function is generally challenging. Using the DTUKF, mapping a point from the PDF with respect to the PDF itself becomes easier [20,30]. Finding a series of unique points, called sigma points, which represent the PDF of the system rather than the PDF of the nonlinear system itself, is not difficult. To further explain this fundamental principle, it is assumed that the mean and covariance of the state vector of a system, denoted by x, are available and represented by x̂ and P, respectively. The nonlinear function that transforms the sigma points is assumed to be $y_i = f(x_i)$. The Discrete-Time Unscented Kalman Filter, due to its use of the nonlinear system itself, has significantly less error in state estimation [31].

By mapping these static vectors through the nonlinear function $y_i = f(x_i)$, the mean and covariance of the transformed vectors, which provide a good approximation of the mean and covariance of y, are obtained. The unscented transformation produces 2n+1 sigma vectors X and appropriately weighted points W. If the state vector of the system and the nonlinear function are assumed to be an n×1 vector and $y_i = f(x_i)$, respectively, we have [30]:

$$
\begin{aligned}
&X_0 = \hat{x} \\
&X_{(i)} = \hat{x} + (\sqrt{(n+\lambda)P})_i^T \quad ,i = 1,\dots,n \\
&X_{(i+n)} = \hat{x} - (\sqrt{(n+\lambda)P})_i^T \quad ,i = 1,\dots,n \\
&W_0^{(m)} = \frac{\lambda}{n+\lambda} \\
&W_0^{(c)} = W_0^{(m)} + (1-\alpha^2+\beta) \\
&W_i^{(m)} = W_i^{(c)} = \frac{1}{2(n+\lambda)} \quad ,i = 1,\dots,2n
\end{aligned}
\tag{7}
$$

In Equation (7), $\sqrt{(n+\lambda)p}_i$ represents the ith row or column of the square root of the matrix $(n+\lambda)p$. $\sqrt{(n+\lambda)p}$ can be obtained using the Cholesky decomposition of the lower triangular matrix. The parameter $\lambda = \alpha^2(n+k) - n$ is a scaling parameter. The parameter α, which determines the spread of the sigma points around the mean x̂, usually has a small positive value within the following range [30]:

$$10^{-4} \le \alpha \le 1 \tag{8}$$

The parameter κ is referred to as the secondary scaling parameter and is typically set to zero. The parameter β is used to incorporate prior knowledge of the distribution of x̂. The weights $W_i^{(m)}$ and $W_i^{(c)}$ are the weights used for the appropriate weighting of the mean and covariance of the ith sigma point, respectively. The sigma vectors are propagated through the nonlinear function f to achieve the transformed sigma points.

$$y_i = f(x_i) \quad , \qquad i = 0, \dots, 2n \tag{9}$$

The mean and covariance of y_i are approximated using a weighted average of the mean and covariance of the transformed sigma points as follows [33]:

$$\begin{aligned} \bar{y}_u &= \sum_{i=0}^{2n} W_i^{(m)} y_i \\ \bar{P}_u &= \sum_{i=0}^{2n} W_i^{(c)} (y_i - \bar{y}_u)(y_i - \bar{y}_u)^T \end{aligned} \tag{10}$$

The implementation of the neutral Kalman filter can be described as follows:

1. Set Transformation by Introducing Each Point Using the Process Model:

$$(\zeta_k^-)_i = f((X_k^-)_i) \quad , \qquad i = 0, \dots, 2n \tag{11}$$

2. Predicted Mean:

$$\hat{x}_k^- = \sum_{i=0}^{2n} W_i^{(m)} (\zeta_k^-)_i \tag{12}$$

3. Predicted Covariance:

$$\bar{P}_k = \sum_{i=0}^{2n} W_i^{(c)} [(\zeta_k^-)_i - \hat{x}_k^-][(\zeta_k^-)_i - \hat{x}_k^-]^T + Q_k \tag{13}$$

4. Prediction of Each Point via the Observation Model:

$$(Z_k^-)_i = h((\zeta_k^-)_i) \tag{14}$$

5. Predicted Observation:

$$\hat{z}_k^- = \sum_{i=0}^{2n} W_i^{(m)} (Z_k^-)_i \tag{15}$$

6. New Covariance:

$$P_{vv} = \sum_{i=0}^{2n} W_i^{(c)} [(Z_k^-)_i - \hat{z}_k^-][(Z_k^-)_i - \hat{z}_k^-]^T + R_k \tag{16}$$

7. Cross Covariance:

$$P_{xz} = \sum_{i=0}^{2n} W_i^{(c)} [(\zeta_k^-)_i - \hat{x}_k^-][(Z_k^-)_i - \hat{z}_k^-]^T \tag{17}$$

8. Update:

$$K_k = P_{xz}{P_{vv}}^{-1}$$
$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - \hat{z}_k^-) \tag{18}$$
$$P_k = {P_k}^- - K_k P_{vv} {K_k}^T$$

To propagate the estimate of the state vector $\hat{x}$ and the error covariance P over time, it is necessary to understand how they change with time, which is dependent on the system model. A fundamental assumption in the Kalman filter is that the time derivative of each state is a linear function of the other states and sources of white noise. Thus, the actual state vector at time t in the Kalman filter is specified by the following relation:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{F}(t)\boldsymbol{x}(t) + \boldsymbol{G}(t)\boldsymbol{w}_s(t) \tag{19}$$

In this relation, $\boldsymbol{w}_s(t)$ is the system noise vector, F(t) is the system matrix, and G(t) is the system noise distribution matrix. The system noise vector comprises independent random noise sources assumed to have a Gaussian distribution with zero mean. The matrices F(t) and G(t) are always known functions. To obtain the estimate of the state vector, the expectation operator is used. The expectation of the actual state vector x(t) is the estimated state vector $\hat{x}(t)$. On the other hand, the expectation of the system noise vector $\boldsymbol{w}_s(t)$ is zero because we consider the noise sources to have a zero mean. Consequently, F(t) and G(t) are known functions and can interchange with the expectation operator, thus applying expectation to (19) we have:

$$E\big(\dot{\boldsymbol{x}}(t)\big) = \frac{\partial}{\partial t}\hat{x}(t) = \boldsymbol{F}(t)\hat{x}(t) \tag{20}$$

By solving the above equation, the state vector estimates at time t as a function of the state estimate at time $t - \tau_s$ is obtained as follows:

$$\hat{x}(t) = \exp\left(\int_{t-\tau_s}^{t} F(t')dt'\right)\hat{x}(t-\tau_s) \tag{21}$$

In the discrete Kalman filter, the state estimate as a linear function of its previous value coupled with the transition matrix $\boldsymbol{\Phi}_{k-1}$ is obtained as follows:

$$\hat{\boldsymbol{x}}_k^- = \boldsymbol{\Phi}_{k-1}\hat{\boldsymbol{x}}_{k-1}^+ \tag{22}$$

Assuming $\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}(t)$ and $\hat{\boldsymbol{x}}_{k-1} = \hat{\boldsymbol{x}}(t-\tau_s)$, the continuous and discrete forms of the Kalman filter are equivalent. Therefore, we have:

$$\boldsymbol{\Phi}_{k-1} \approx \exp(\boldsymbol{F}_{k-1}\tau_s) \tag{23}$$

Assuming the data is only available at times $t - \tau_s$ and t, the system matrix $\boldsymbol{F}_{k-1}$ can be obtained as $\frac{1}{2}\big(F(t-\tau_s) + F(t)\big)$ or by averaging the individual parameters of F at times $t - \tau_s$ and t. In general, relation (23) cannot be directly computed because each element of the matrix exponent differs from the matrix exponential. Therefore, numerical methods must be employed. One of these methods involves calculating the transition matrix using the power series expansion for the matrix exponential over the time interval τ_s:

$$\boldsymbol{\Phi}_{k-1} = \sum_{r=0}^{\infty} \frac{F_{k-1}^r \tau_s^r}{r!} = I + \boldsymbol{F}_{k-1}\tau_s + \cdots \tag{24}$$

In this case, the system matrix and the noise matrix of the Kalman filter can be considered as follows:

$$\boldsymbol{F}^n = \begin{bmatrix} \boldsymbol{F}_{11} & \boldsymbol{F}_{12} \\ \boldsymbol{F}_{21} & \boldsymbol{F}_{22} \end{bmatrix}, \quad \boldsymbol{Q} = \begin{bmatrix} n_a^2\boldsymbol{I}_3 & 0_3 \\ 0_3 & 0_3 \end{bmatrix}\tau_s \tag{25}$$

4. GENETIC ALGORITHM

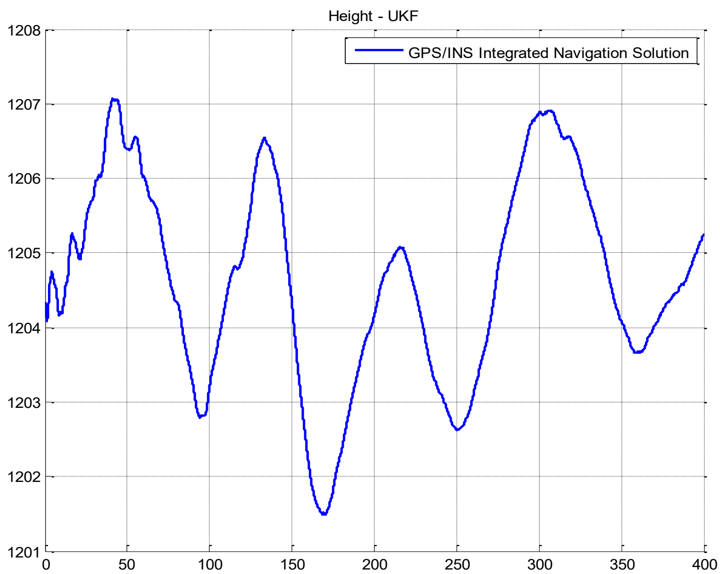
Genetic algorithms are a type of stochastic search algorithm inspired by nature, widely used in solving engineering optimization problems. They are employed to find optimal relationships for prediction or pattern matching. These algorithms are particularly efficient for discrete, nonlinear, and noisy systems [32]. This paper, for the first time, explores obtaining the optimal point for noise matrices Q and R in the Kalman filter by introducing a suitable function to evaluate numerical hypotheses. The goal is to achieve convergence of the mean outputs to the reference value with minimal variance [33]. Equation (26) presents the suitable function used in this study:

$$e(v_{ij}) = f(S_k^{v_{ij}}) - f(S_k) \tag{26}$$

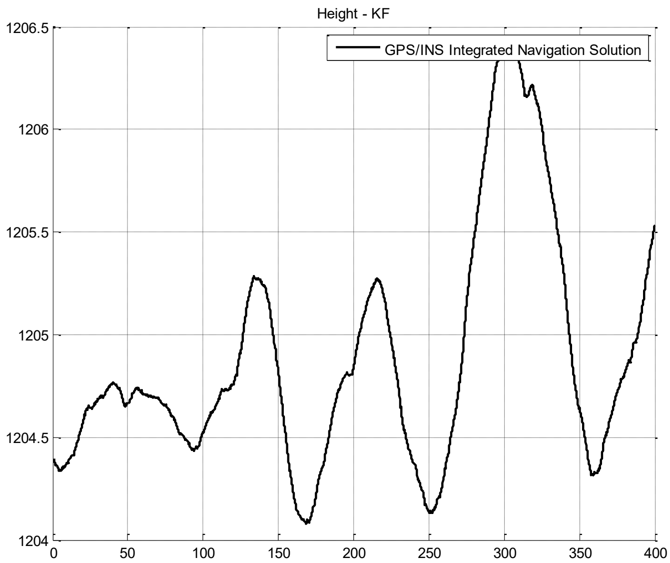
where $f(S_k)$ and $f(S_k^{v_{ij}})$ are the parents and the offspring, respectively, and the best generation is obtained based on the random $e(v_{ij})$. Notably, this is the first time that a genetic optimization approach has been proposed in this context. It is essential to note that to calculate the optimal values of the Q and R matrices using the genetic algorithm, an initial value is required. Thus, by introducing the elements of these two matrices, the initial value is experimentally written. Subsequently, by introducing the initial value as the system input, the cost function, which needs to be minimized, is defined as the system output. Finally, by obtaining the optimal values for the Q and R matrices, the desired outcome for error reduction over time using the discrete-time neutral Kalman filter will be achieved.

5. COMPARISON OF THE PERFORMANCE OF LINEAR AND NEUTRAL KALMAN FILTERS IN NONLINEAR INERTIAL NAVIGATION SYSTEMS

This section focuses on designing and simulating the mentioned filters on a real nonlinear inertial navigation system, analyzing their performance based on the results obtained. The system used as the primary process for designing the linear and discrete-time neutral Kalman filters is the integrated inertial navigation system. Considering the integrated navigation system, the inputs are acceleration and angular velocity, while the outputs for optimization are position, velocity in the north, south, and east directions, and altitude. After obtaining the optimal points and placing them in the noise matrices Q and R, the system outputs, including position, velocity in the north, south, and east directions, and altitude, are displayed using both the linear and neutral Kalman filters. A comparison of these outputs demonstrates the superior performance of the neutral Kalman filter. The results depict data from the integrated navigation system over 400 seconds at a frequency of 10 Hz.



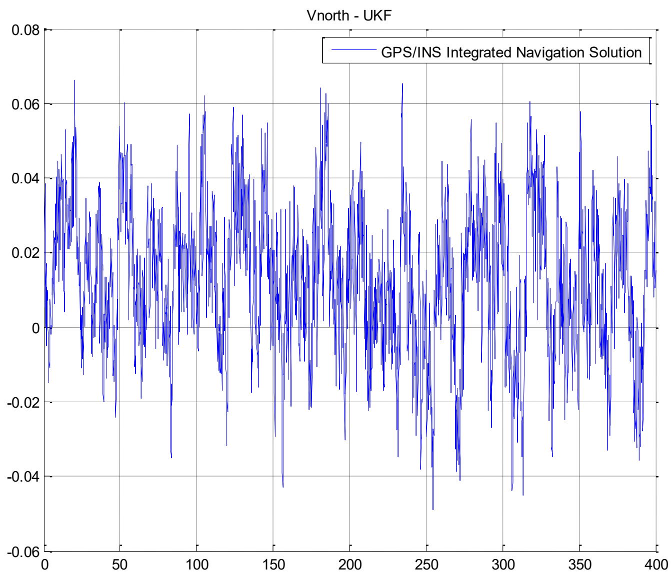
(a)



(b)

Fig. 2. Comparison of Altitude Output (in meters) from the Integrated Navigation System Using: (a) Neutral Kalman Filter and (b) Linear Kalman Filter

Observing Figure 2, which shows the altitude output for the integrated navigation system using both the linear and neutral Kalman filters, it can be stated that the mean altitude output in the neutral Kalman filter decreases relative to the system's reference value of 4.1204. However, the variance increases. The variance does not affect the system's error; hence, with the decrease in the mean value, the system error also decreases, indicating the superior performance of this filter compared to the linear Kalman filter.



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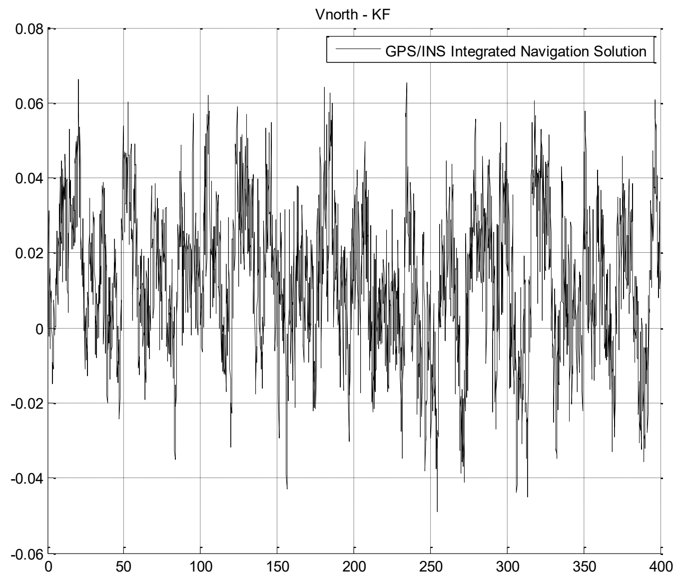
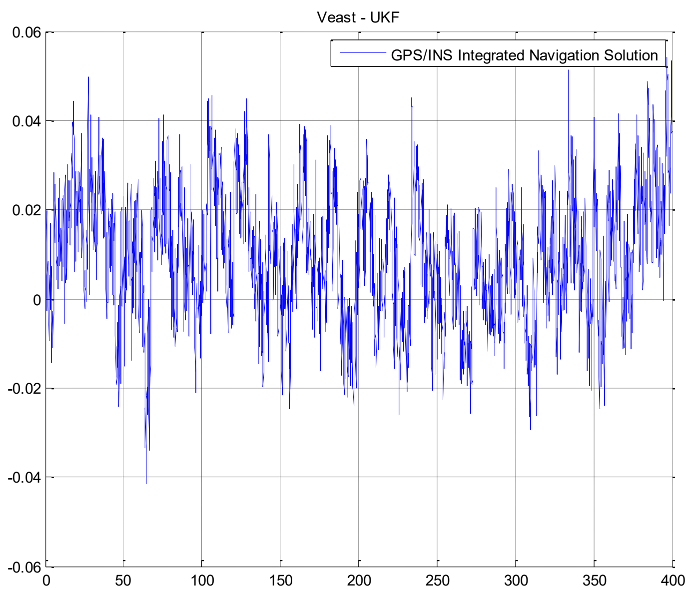


(b)

Fig. 3. Comparison of Velocity Output in the North Direction (in meters per second) from the Integrated Navigation System Using: (a) Neutral Kalman Filter and (b) Linear Kalman Filter

In Figure 3, it is observed that using the discrete-time neutral Kalman filter, the variance and mean value for convergence to the reference value, which is zero, increase compared to the linear Kalman filter. This indicates an increase in error around the reference value. Consequently, the linear Kalman filter performs better for this output.



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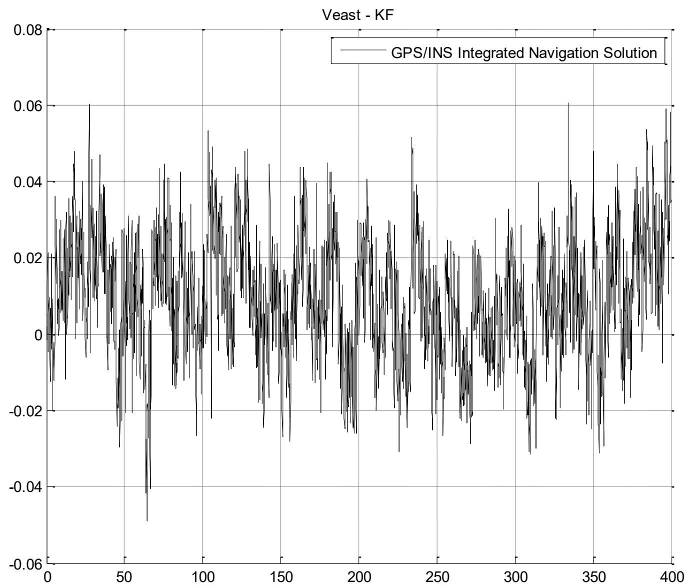
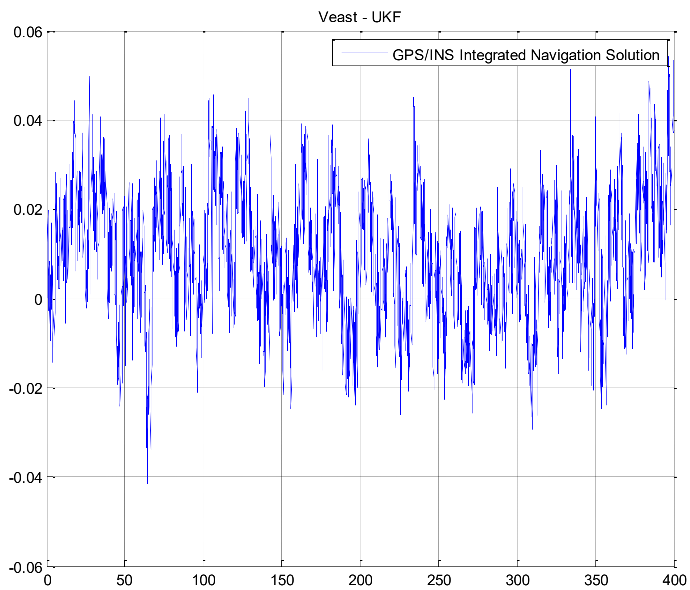


(b)

Fig. 4. Comparison of Velocity Output in the South Direction (in meters per second) from the Integrated Navigation System Using: (a) Neutral Kalman Filter and (b) Linear Kalman Filter

In Figure 4, the use of the neutral Kalman filter reduces the variance around the reference value and also decreases the mean value, thereby improving convergence to the desired value. Thus, the velocity output in the south direction has less error compared to the linear Kalman filter, indicating the superior performance of the neutral Kalman filter for this output.



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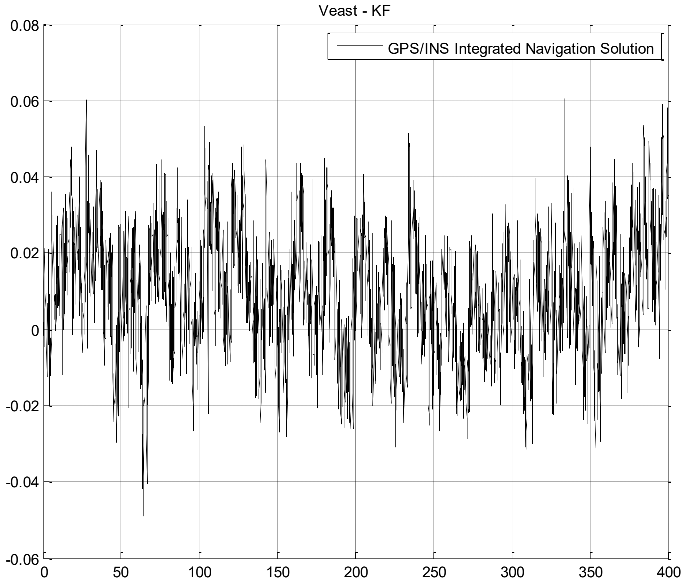
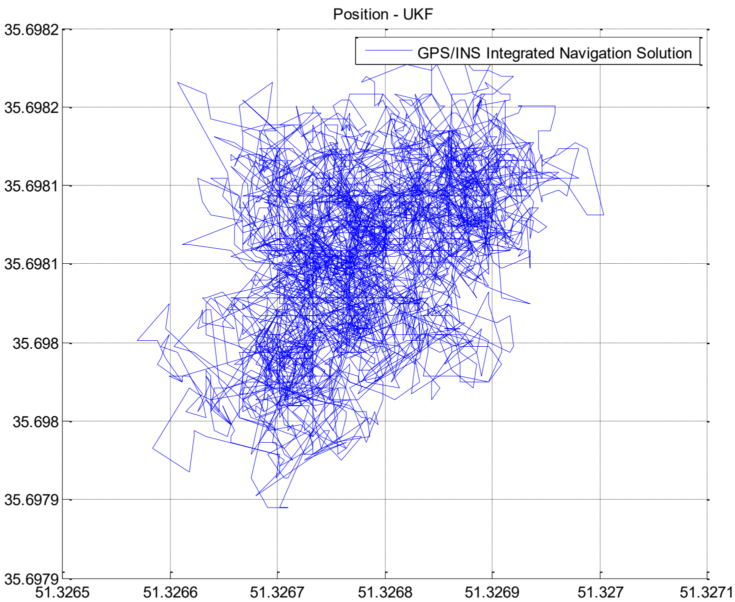


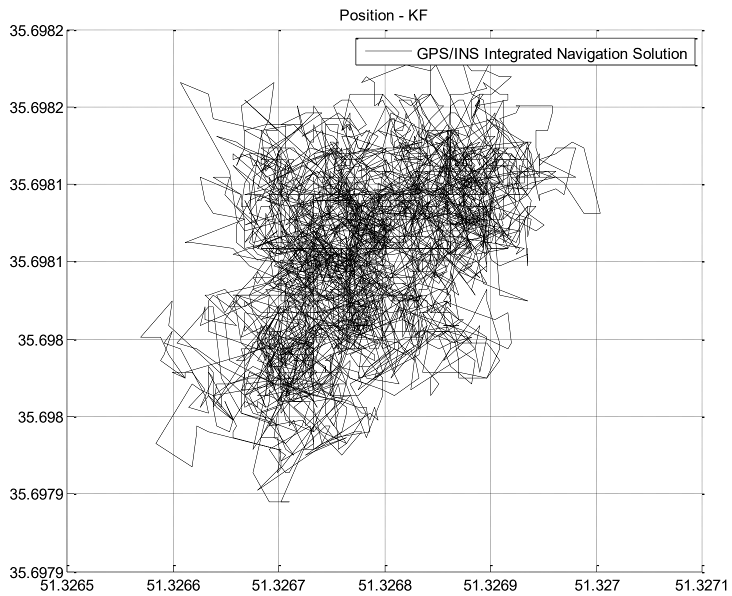
(b)

Fig. 5. Comparison of Velocity Output in the East Direction (in meters per second) from the Integrated Navigation System Using: (a) Neutral Kalman Filter and (b) Linear Kalman Filter

Observing Figure 5, it can be concluded that using the linear Kalman filter slightly reduces the error in the mean output but increases the variance. Since the variance does not affect the system error, the reduction in the mean decreases the error, indicating that the linear Kalman filter performs better than the neutral Kalman filter for this output.



(a)



(b)

Fig. 6. Comparison of Latitude Output versus Longitude Output (in degrees) from the Integrated Navigation System Using: (a) Neutral Kalman Filter and (b) Linear Kalman Filter

In Figure 6, using the discrete-time neutral Kalman filter and comparing it with the linear Kalman filter, it is observed that both filters perform equally well in terms of mean value. However, the variance in the position output is significantly reduced using the neutral Kalman filter compared to the ordinary Kalman filter, indicating the better and more appropriate performance of the discrete-time unscented Kalman filter (DTUKF). To further evaluate the performance of the integrated system using the discrete-time neutral Kalman filter compared to the linear Kalman filter, the criterion of the sum of the mean square errors and output variances is used, as shown in Tables (1) and (2).

Table 1. Mean and Variance Values of Integrated System Outputs Using the Neutral Kalman Filter Relative to the Reference Value

Neutral Kalman filter			
Variance	**Value**	**Mean**	**Value**
Altitude	1.8672	Altitude	$1.2046e^{+3}$
Southward Speed	$3.1078e^{-4}$	Southward Speed	-0.0034
Northward Speed	$3.6605e^{-4}$	Northward Speed	0.0127
Eastward Speed	$1.9727e^{-4}$	Eastward Speed	0.0096
Position	$2.6498e^{-9}$	Position	35.6981

Table 2. Mean and Variance Values of Integrated System Outputs Using the Linear Kalman Filter Relative to the Reference Value

Linear Kalman filter			
Variance	**Value**	**Mean**	**Value**
Altitude	0.1098	Altitude	$1.2050e^{+3}$
Southward Speed	$7.2415e^{-4}$	Southward Speed	-0.0036
Northward Speed	0.0340	Northward Speed	0.1228
Eastward Speed	$9.3314e^{-4}$	Eastward Speed	0.0079
Position	$2.7965e^{-9}$	Position	35.6981

From the above tables, it is observed that the mean altitude output of the system using the discrete-time neutral Kalman filter has an error of 0.3 meters less than that of the ordinary Kalman filter, indicating a significant improvement in mean convergence to the reference mean value. Additionally, while the velocity output in some directions using the neutral Kalman filter has slightly increased compared to the ordinary Kalman filter, it remains oscillating around the reference value with excellent convergence, indicating minimal system error. Considering the increase in variance in the integrated system with the neutral Kalman filter due to the nonlinearity of the system and the more realistic performance of this filter compared to the ordinary Kalman filter, all existing noises in the system are examined, leading to an increase in variance, which does not imply an increase in system error. Ultimately, the mean values of the integrated system outputs indicate a significant reduction in integrated navigation error over time compared to the inertial navigation system.

6. CONCLUSION

This paper, for the first time, proposed and designed the use of a discrete-time neutral Kalman filter optimized by a genetic algorithm for estimation in nonlinear systems. The linear Kalman filter family is one of the best minimum variance optimal estimation tools for implementation on linear systems. However, implementing this type of filter on nonlinear systems, due to the assumption of system linearity and neglecting nonlinear dynamics, results in unacceptable estimation error variance leading to divergence. The improved version of the linear Kalman filter, the extended Kalman filter, has significant applications in practical nonlinear systems. However, its main drawback is its heavy reliance on the predefined dynamic model and the use of the Jacobian matrix for first-order system linearization, reducing the accuracy of nonlinear system parameter estimation. Therefore, this paper proposed the design and use of the unscented Kalman filter (UKF) as a nonlinear estimator.

The superiority of the UKF over the extended Kalman filter is not only theoretical but also practical. This filter approximates the nonlinear distribution using a limited number of carefully chosen sigma points to propagate the state distribution. Unlike the extended Kalman filter, which uses the Jacobian matrix for first-order accuracy, the UKF calculates a minimal set of sigma points and achieves at least second-order accuracy in mean and covariance, significantly enhancing estimation performance. Furthermore, as another innovation, a genetic algorithm was employed to obtain optimal points in the system noise covariance matrix Q and the measurement noise covariance matrix R, increasing confidence in the integrated navigation system's performance. This algorithm was used to achieve the least variance in reference navigation values and the highest convergence of mean response during navigation. Simulation results demonstrated that implementing and comparing the linear Kalman filter and the genetic algorithm-optimized UKF on the integrated navigation system, a nonlinear system with complex dynamics, showed significantly improved performance and accuracy of the discrete-time UKF compared to the linear Kalman filter. This indicates a substantial increase in navigation and estimation accuracy with minimal error in speed, position, and altitude.

Transparency Statement

The data supporting this study are available upon reasonable request to the corresponding author, subject to ethical and confidentiality considerations.

Acknowledgments

We would like to express our gratitude to all individuals who contributed to this project.

Declaration of Interest

The authors declare that they have no competing interests.

Funding

This research received no specific grant from any funding agency, commercial, or not-for-profit sectors.

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