



## Type 2 Neuro-Fuzzy Controller for a Class of Delayed Nonlinear Systems

K. Sabahi<sup>1,\*</sup> 

<sup>1</sup> Assistant Professor, Department of Engineering Sciences, Faculty of New Technologies, University of Mohaghegh Ardabili, Namin, Iran

ARTICLE INFO	ABSTRACT
<p>Article History:            Received 3 May 2022            Received in revised form 11 July 2022            Accepted 3 September 2022            Available online 5 September 2022</p>	<p>This article explores the design of a type 2 neuro-fuzzy controller specifically for managing delayed nonlinear systems through feedback error training. The feedback error training framework incorporates a traditional controller in the feedback loop to stabilize the system. Meanwhile, the forward loop employs a type 2 neuro-fuzzy controller, which serves as an intelligent controller to address system nonlinearity and time delay issues. The parameters of the type 2 neuro-fuzzy controller are fine-tuned using the gradient descent method within this framework. To assess the stability of both the closed-loop system and the parameter adjustment algorithm, a Lyapunov-Krasowski function is utilized. This function demonstrates that the tracking error can be reduced to zero, even with the presence of delay in the control system input. Furthermore, the regulation rules for the intelligent controller's parameters can be derived without needing the exact mathematical model or parameters of the system being controlled, thus simplifying the calculations. The proposed method has been applied to control an inverse pendulum system characterized by nonlinear behavior and time-varying delays in its input due to network-based control. Additionally, sensor measurements are assumed to be noisy. Simulation results validate the effectiveness of the designed controller across various time delay scenarios and noise levels.</p>
<p>Keywords:            Delayed Nonlinear Systems, Type 2 Neuro-Fuzzy Controller, Lyapunov-Krasovski Function, Measurement Noise, Stability</p>	

### 1. INTRODUCTION

Modern industrial and practical systems often face issues with time delay [1-9], which can not only reduce system efficiency but, in some cases, lead to system instability [10]. Examples of such systems include chemical reactors, thermal systems, long transmission lines, and power systems [11]. In industrial applications, methods such as Ziegler-Nichols [12] and Smith predictor [13] were among the first to address time delay in controller parameter design, thus improving closed-loop behavior. These methods require an accurate system model and assume a constant time delay, which limits their applicability to systems with nonlinear dynamic models and variable time delays. It is worth noting that dynamic modeling of delayed systems is typically expressed in three forms: input delay, state delay, and a combination of both (input and state delays), and controllers must be designed accordingly. The Lyapunov-Krasovski stability theorem is a method used in the time domain to analyze the stability of delayed systems and design controllers [14]. In applying the Lyapunov-Krasovski stability theorem, which is also applicable

\* Corresponding Author: [k.sabahi@uma.ac.ir](mailto:k.sabahi@uma.ac.ir)

Assistant Professor, Department of Engineering Sciences, Faculty of New Technologies, University of Mohaghegh Ardabili, Namin, Iran



to uncertain systems with variable time delays, the final result involves solving a Linear Matrix Inequality (LMI) [15]. Depending on how the Lyapunov-Krasovski function is defined, the resulting LMI can be either dependent on or independent of the time delay [16]. This theorem also allows for the analysis of robust stability and the design of robust controllers. References [17-18] discuss the design of  $H_\infty$  robust controllers for uncertain systems with variable time delays. A major challenge with these methods is the lack of a general solution for controlling nonlinear delayed systems and the need for an accurate system model [19]. In contrast to classical methods, intelligent control methods such as neural networks and fuzzy systems have been used for controlling nonlinear delayed systems. References [20-21] exploit the capability of neural networks for estimating uncertain delayed nonlinear functions. In these methods, the nonlinear system is approximated within a closed set, and this approximation is simultaneously used to obtain the control signal. These methods, also known as indirect control methods, use a separate neural network for approximating each nonlinear function, which poses challenges in online applications.

The Takagi-Sugeno (TS) fuzzy system modeling approach is also used to convert delayed nonlinear systems into linear systems (locally) and then employs robust control techniques such as  $H_\infty$  for designing controllers for TS fuzzy systems [22-23]. The TS fuzzy system modeling approach helps overcome the limitations of classical methods when applied to nonlinear systems. To address uncertainties and measurement noise, the Type 2 fuzzy system, an extension of the Type 1 fuzzy system, is employed [24]. The TS fuzzy system-based control method is offline and requires the measurement of all state variables, which presents issues in physical systems. Additionally, Reference [25] uses a Type 2 neuro-fuzzy controller with a gradient descent algorithm for controlling delayed systems. Although this article proves system stability using the Lyapunov method, it overlooks the effect of delay on stability analysis and parameter adjustment. The combination of classical and intelligent controllers, where the classical controller handles closed-loop stability and the intelligent controller addresses nonlinearity and time delay, is also a focus of research [26]. This combined approach, also known as Feedback Error Learning (FEL), has been applied to time-invariant delayed Linear Time-Invariant (LTI) systems where the time delay is considered known and fixed [27].

To enhance the performance of the error back-propagation method, a Type 2 neuro-fuzzy network (T2FNN) has been used instead of a neural network for controlling systems without delay [28]. Considering these aspects and the prevalence of delay issues in practical systems, this paper uses a Feedback Error Learning method where a Type 2 neuro-fuzzy controller is used in the forward loop and a Proportional-Derivative (PD) controller is employed in the feedback loop to control a class of delayed nonlinear systems. The gradient descent method is used to adjust the parameters of the Type 2 neuro-fuzzy controller. A suitable Lyapunov-Krasovski function is employed to analyze the stability of the closed-loop system, taking into account the mentioned adjustment algorithm. In this method, the effect of time delay on parameter adjustment is considered, which improves the performance and stability of the closed-loop system. The proposed method is applied to control an inverted pendulum system, which experiences input delays due to network control. The introduced controller is compared with a structure from Reference [29] that uses two Type 1 fuzzy systems for approximating nonlinear functions, and its effectiveness is demonstrated against various delay and measurement noise levels. The structure of the paper is as follows: Section 2 describes the system specifications, Section 3 details the proposed control structure and closed-loop stability analysis, Section 4 presents simulations, and Section 5 concludes the study.

## 2. SYSTEM SPECIFICATIONS

Consider a class of nonlinear systems with time delays in the input, described as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \vdots \\ \dot{x}_n = f(x) + g(x)u(t - \tau(t)) + d(t) \end{cases} \tag{1}$$

where  $x \in \mathbb{R}^n$  represents the state variables,  $\tau(t) > 0$  denotes the time delay in the system's input,  $u$  is the input signal, and  $f$  and  $g$  are nonlinear functions. Additionally,  $d(t)$  is a disturbance signal that is bounded. In this system, it is assumed that within the interval  $t \in [-\tau, 0]$ , the signal  $u$  is zero and the time delay  $\tau$  satisfies the following condition:

$$0 \leq \tau(t) \leq \tau_{\max}, 0 \leq \dot{\tau}(t) < 1 \tag{2}$$

where  $\tau_{\max}$  is a known upper bound on the time delay.

### 3. PROPOSED CONTROLLER STRUCTURE

The proposed control structure is depicted in Figure 1.

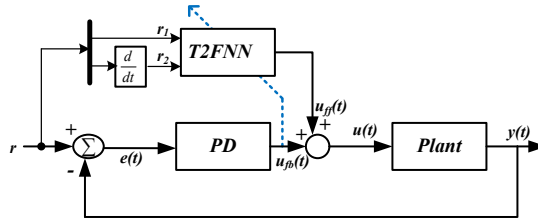


Fig. 1. Proposed Control Structure

As shown, this structure consists of a classic PD controller in the feedback path and a Type-2 Neuro-Fuzzy Controller in the feedforward path. The PD controller plays a crucial role in system stability, handling transient dynamics by training the feedforward controller, which subsequently takes over in the steady-state mode. It is important to note that when the feedforward controller takes over system control, an inverse system is constructed and the reference tracking problem is addressed [25].

Moreover, in this structure, unlike the direct method where the tracking error  $e$  is used to train the parameters of the Type-2 Neuro-Fuzzy Controller, the output of the classic controller ( $u_{fb}$ ) is used for tuning its parameters. Therefore, if the signal  $u_{fb}$  approaches zero, the Type-2 Neuro-Fuzzy Controller assumes control of the system, and control objectives such as regulation and reference input tracking  $r$  are achieved. Consequently, the training signal for adjusting the parameters of the feedforward controller is defined as follows:

$$e_{tr}(t) = u_{fb}(t) = u(t) - u_{ff}(t) \tag{3}$$

Where  $u$ ,  $u_{ff}$ , and  $e_{tr}$  represent the final control signal, the output of the Type-2 Neuro-Fuzzy Controller, and the output of the PD controller (training signal), respectively. The output of the PD controller can be expressed as:

$$u_{fb}(t) = k_p e(t) + k_d \dot{e}(t) \tag{4}$$

where  $k_p$  and  $k_d$  are positive, adjustable parameters of the controller. It is noteworthy that this choice of controller is due to its effectiveness in improving transient performance and stability compared to other classic controllers like Proportional-Integral (PI) or Proportional (P) controllers. In the proposed control structure, the following assumptions are made regarding the final control signal:

$$|u(t)| < \gamma, |\dot{u}(t)| < \beta \tag{5}$$

where  $\gamma$  and  $\beta$  are known parameters.

#### 3.1. Type-2 Neuro-Fuzzy Controller Structure

The Type-2 Neuro-Fuzzy Controller has two inputs: the error and its derivative, and one output  $u_{ff}$ . The membership functions for the antecedent part are Type-2 fuzzy sets, while the membership function for the consequent part is a fixed number. This neural-fuzzy network structure is called A2-C0 [30].

The j -th rule for the Takagi-Sugeno-Kang (TSK) zero-order model is given by:

$$R^j : \text{if } r_1(t) \text{ is } \tilde{A}_{j1} \text{ and } r_2(t) \text{ is } \tilde{A}_{j2}, \text{ Then } y_j = c_j \tag{6}$$

where  $A_{ji}$  represents the Type-2 membership function for the j-th rule and the i-th input, and  $r_1 = r(t)$  and  $r_2 = \dot{r}(t)$  are inputs to the Type-2 Neuro-Fuzzy Controller. Additionally,  $c_j$  represents the adjustable parameters of the consequent part. The membership function for the antecedent part is considered to be a Gaussian function with uncertainty in the standard deviation, as shown in Figure 2. Without loss of generality, the lower and upper bounds of the j-th rule are defined as follows:

$$\underline{f}_j = \underline{\mu}^{j1}(r_1) * \underline{\mu}^{j2}(r_2) \tag{7}$$

$$\bar{f}_j = \bar{\mu}^{j1}(r_1) * \bar{\mu}^{j2}(r_2) \tag{8}$$

where T-norm is considered as multiplication in this paper. The lower and upper values of the membership function (according to Figure 2), using the Gaussian function, are expressed as:

$$\underline{\mu}^{j1}(r_1) = \exp\left(-\frac{1}{2}\left(\frac{r_1 - m^{j1}}{\sigma_1^{j1}}\right)^2\right) \tag{9}$$

$$\bar{\mu}^{j1}(r_1) = \exp\left(-\frac{1}{2}\left(\frac{r_1 - m^{j1}}{\sigma_2^{j1}}\right)^2\right) \tag{10}$$

$$\underline{\mu}^{j2}(r_2) = \exp\left(-\frac{1}{2}\left(\frac{r_2 - m^{j2}}{\sigma_1^{j2}}\right)^2\right) \tag{11}$$

$$\bar{\mu}^{j2}(r_2) = \exp\left(-\frac{1}{2}\left(\frac{r_2 - m^{j2}}{\sigma_2^{j2}}\right)^2\right) \tag{12}$$

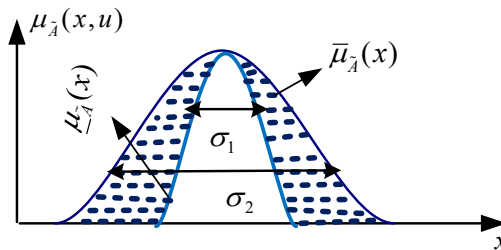


Fig. 2. Type-2 Membership Function with Uncertainty in Standard Deviation

where  $\sigma_1, \sigma_2, \sigma_m$  are the tunable parameters of the fuzzy rule base. The final output of the Type-2 TSK (Takagi-Sugeno-Kang) neuro-fuzzy network can be expressed as follows.

$$u_{ff}(t) = \frac{\sum_{j=1}^M c_j (f_j + \bar{f}_j)}{\sum_{j=1}^M (f_j + \bar{f}_j)} \quad (13)$$

This output, after normalization, can also be expressed in a simpler form as follows:

$$u_{ff}(t) = \sum_{j=1}^M c_j \phi_j \quad (14)$$

where:

$$\phi_j = \frac{f_j + \bar{f}_j}{\sum_{j=1}^M (f_j + \bar{f}_j)} = \frac{f_j}{\sum_{j=1}^M f_j} \quad (15)$$

### 3.2. Tuning of Type-2 Neuro-Fuzzy Controller Parameters

The error backpropagation method is a derivative-based approach used for online tuning of neuro-fuzzy network controller parameters. In this method, the cost function that needs to be minimized is defined as follows:

$$E = \frac{1}{2} \sum (e_r)^2 = \frac{1}{2} \sum (u - u_{ff})^2 \quad (16)$$

As previously mentioned, minimizing this function will achieve the reference signal tracking. In this approach, the error E must be computed and used to adjust and update the parameters of the Type-2 neuro-fuzzy controller. For this purpose, the gradient descent algorithm is used to update the controller parameters as follows [25]:

$$m^{ji}(t+1) = \alpha m^{ji}(t) + \eta(t) \frac{\partial E(t)}{\partial m^{ji}(t)} \quad (17)$$

$$\sigma_1^{ji}(t+1) = \alpha \sigma_1^{ji}(t) + \eta(t) \frac{\partial E(t)}{\partial \sigma_1^{ji}(t)} \quad (18)$$

$$\sigma_2^{ji}(t+1) = \alpha \sigma_2^{ji}(t) + \eta(t) \frac{\partial E(t)}{\partial \sigma_2^{ji}(t)} \quad (19)$$

$$c^j(t+1) = \alpha c^j(t) + \eta(t) \frac{\partial E(t)}{\partial c^j(t)} \quad (20)$$

Additionally, the stability of the closed-loop system for the delayed system is also discussed using the mentioned Lyapunov-Krasovskii function.

### 3.3. Stability Analysis of the Closed-Loop System

Considering the derivative of the parameters of the Type-2 Neuro-Fuzzy Controller as expressed below (for simplicity, the argument t in the functions is omitted):

$$\dot{m}^{ji} = (\alpha - 1)m^{ji} + \eta \frac{\partial E}{\partial m^{ji}} \quad (21)$$

$$\dot{\sigma}_1^{ji} = (\alpha - 1)\sigma_1^{ji} + \eta \frac{\partial E}{\partial \sigma_1^{ji}} \quad (22)$$

$$\dot{\sigma}_2^{ji} = (\alpha - 1)\sigma_2^{ji} + \eta \frac{\partial E}{\partial \sigma_2^{ji}} \quad (23)$$

$$\dot{c}^j = (\alpha - 1)c^j + \eta \frac{\partial E}{\partial c^j} \tag{24}$$

The derivative of the function E with respect to the parameters can be expressed using the chain rule as follows:

$$\frac{\partial E}{\partial m^{ji}} = \frac{\partial E}{\partial e_{ir}} \frac{\partial e_{ir}}{\partial u_{ff}} \frac{\partial u_{ff}}{\partial m^{ji}} = (e_{ir})(-1)\left(\frac{\partial u_{ff}}{\partial m^{ji}}\right) \tag{25}$$

Similarly, for other parameters, we have:

$$\begin{aligned} \frac{\partial E}{\partial \sigma_1^{ji}} &= (e_{ir})(-1)\left(\frac{\partial u_{ff}}{\partial \sigma_1^{ji}}\right) \\ \frac{\partial E}{\partial \sigma_2^{ji}} &= (e_{ir})(-1)\left(\frac{\partial u_{ff}}{\partial \sigma_2^{ji}}\right) \\ \frac{\partial E}{\partial c^j} &= (e_{ir})(-1)\left(\frac{\partial u_{ff}}{\partial c^j}\right) \end{aligned} \tag{26}$$

The derivative of the output of the Type-2 Neuro-Fuzzy Controller  $u_{ff}$  can be expressed as:

$$\begin{aligned} \dot{u}_{ff} &= \frac{\partial u_{ff}}{\partial m_1^{ji}} \dot{m}_1^{ji} + \frac{\partial u_{ff}}{\partial \sigma_1^{ji}} \dot{\sigma}_1^{ji} + \\ &\quad \frac{\partial u_{ff}}{\partial \sigma_2^{ji}} \dot{\sigma}_2^{ji} + \frac{\partial u_{ff}}{\partial c^j} \dot{c}^j + \frac{\partial u_{ff}}{\partial r} \dot{r} \end{aligned} \tag{27}$$

where  $r$  is the input to the controller. Now, the Lyapunov-Krasovskii function is defined as:

$$V(t) = \frac{1}{2} e_{ir}(t)^2 + \int_{t-\tau(t)}^t e_{ir}(s)^2 ds + \int_{t-\tau(t)}^t \dot{e}_{ir}(s)^2 ds + \int_{-\tau_{max}}^0 \int_{t+\theta}^t e_{ir}(\theta)^2 d\theta ds + \frac{1}{2\gamma} (\eta - \eta^*)^2 \tag{28}$$

where  $\gamma$  and  $\eta^*$  are the design parameters and desired learning rate, respectively. Taking the derivative of the above function, we get:

$$\dot{V}(t) = e_{ir} \dot{e}_{ir} + e_{ir}^2 - (1 - \dot{\tau}) e_{ir} (t - \tau)^2 + \dot{e}_{ir}^2 - (1 - \dot{\tau}) \dot{e}_{ir} (t - \tau)^2 + \tau_{max} e_{ir}(t)^2 - \int_{t+\theta}^t e_{ir}(s)^2 ds + \frac{\dot{\eta}}{\gamma} (\eta - \eta^*) \tag{29}$$

Considering assumptions (2) and (5), this can be written as:

$$\dot{V}(t) \leq e_{ir} \dot{e}_{ir} + e_{ir}^2 + \dot{e}_{ir}^2 + \tau_{max} e_{ir}^2 + \frac{\dot{\eta}}{\gamma} (\eta - \eta^*) < e_{ir} (\beta - \dot{u}_{ff}) + (1 + \tau_{max}) e_{ir}^2 + \dot{e}_{ir}^2 + \frac{\dot{\eta}}{\gamma} (\eta - \eta^*) \tag{30}$$

Substituting from equation (27), we obtain:

$$\dot{V}(t) < e_{ir} \left( \beta - \frac{\partial u_{ff}}{\partial m_1^{ji}} \dot{m}_1^{ji} - \frac{\partial u_{ff}}{\partial \sigma_1^{ji}} \dot{\sigma}_1^{ji} - \frac{\partial u_{ff}}{\partial \sigma_2^{ji}} \dot{\sigma}_2^{ji} - \frac{\partial u_{ff}}{\partial c^j} \dot{c}^j - \frac{\partial u_{ff}}{\partial r} \dot{r} \right) + (1 + \tau_{max}) e_{ir}^2 + \dot{e}_{ir}^2 + \frac{\dot{\eta}}{\gamma} (\eta - \eta^*) \tag{31}$$

Substituting equations (25) and (26) into the above relation gives:

$$\dot{V}(t) < |e_{ir}| \beta + \frac{\partial E}{\partial m_1^{ji}} \dot{m}_1^{ji} + \frac{\partial E}{\partial \sigma_1^{ji}} \dot{\sigma}_1^{ji} + \frac{\partial E}{\partial \sigma_2^{ji}} \dot{\sigma}_2^{ji} + \frac{\partial E}{\partial c^j} \dot{c}^j - e_{ir} \frac{\partial u_{ff}}{\partial r} \dot{r} + (1 + \tau_{max}) e_{ir}^2 + \dot{e}_{ir}^2 + \frac{\dot{\eta}}{\gamma} (\eta - \eta^*) \tag{32}$$

Substituting the derivatives of the Type-2 Neuro-Fuzzy Controller parameters from equations (21) to (24) into the above equation, we have:

$$\begin{aligned} \dot{V}(t) < |e_r| \beta + \frac{\partial E}{\partial m_1^{ji}} ((\alpha - 1)m^{ji} + \eta \frac{\partial E}{\partial m^{ji}}) + \frac{\partial E}{\partial \sigma_1^{ji}} ((\alpha - 1)\sigma_1^{ji} + \eta \frac{\partial E}{\partial \sigma_1^{ji}}) + \frac{\partial E}{\partial \sigma_2^{ji}} ((\alpha - 1)\sigma_2^{ji} + \eta \frac{\partial E}{\partial \sigma_2^{ji}}) + \\ \frac{\partial E}{\partial c^j} ((\alpha - 1)c^j + \eta \frac{\partial E}{\partial c^j}) - e_r \frac{\partial u_{ff}}{\partial r} \dot{r} + (1 + \tau_{\max})e_r^2 + \dot{e}_r^2 + \frac{\dot{\eta}}{\gamma}(\eta - \eta^*) \end{aligned} \quad (33)$$

By choosing  $\eta^* > \beta$  and:

$$\dot{\eta} = \gamma |e_r| \quad (34)$$

The above relation, after rearrangement, can be written as:

$$\begin{aligned} \dot{V}(t) < - \overbrace{(\eta \frac{\partial E}{\partial \sigma_1^{ji}})^2 + (1 - \alpha)\sigma_1^{ji} \frac{\partial E}{\partial \sigma_1^{ji}} - 0.5\eta |e_r|}^I - \overbrace{(\eta \frac{\partial E}{\partial \sigma_2^{ji}})^2 + (1 - \alpha)\sigma_2^{ji} \frac{\partial E}{\partial \sigma_2^{ji}} - 0.5\eta |e_r|}^{II} \\ - \overbrace{(\eta \frac{\partial E}{\partial m^{ji}})^2 + (1 - \alpha)m^{ji} \frac{\partial E}{\partial m^{ji}} - \dot{e}_r^2}^{III} - \overbrace{(\eta \frac{\partial E}{\partial c^j})^2 + (1 - \alpha)c^j \frac{\partial E}{\partial c^j} - (1 + \tau_{\max})e_r^2 + e_r \frac{\partial u_{ff}}{\partial r} \dot{r}}^{IV} \end{aligned} \quad (35)$$

The derivative of the Lyapunov-Krasovskii function ( $\dot{V}(t)$ ) will be strictly negative if the following equations exist:

$$\dot{V}(t) < - \overbrace{(\frac{\partial E}{\partial \sigma_1^{ji}} + A)^2}^I - \overbrace{(\frac{\partial E}{\partial \sigma_2^{ji}} + B)^2}^{II} - \overbrace{(\frac{\partial E}{\partial m^{ji}} + C)^2}^{III} - \overbrace{(\frac{\partial E}{\partial c^j} + D)^2}^{IV} \leq 0 \quad (36)$$

$$\begin{aligned} ((1 - \alpha)\sigma_1^{ji})^2 - 4(-0.5\eta |e_r|)(-\eta) &= 0 \\ ((1 - \alpha)\sigma_2^{ji})^2 - 4(-0.5\eta |e_r|)(-\eta) &= 0 \\ ((1 - \alpha)m^{ji})^2 - 4(-\dot{e}_r^2)(-\eta) &= 0 \\ ((1 - \alpha)c^j)^2 - 4(-(1 + \tau_{\max})e_r^2 + e_r \frac{\partial u_{ff}}{\partial r} \dot{r})(-\eta) &= 0 \end{aligned} \quad (37)$$

Therefore, the roots of equation (35) can be computed as:

$$A_1^1 = \frac{(1 - \alpha)A^1}{2\eta}, A_2^1 = \frac{(1 - \alpha)A^2}{2\eta} \quad (38)$$

$$B_1^1 = \frac{(1 - \alpha)B^1}{2\eta}, A_2^1 = \frac{(1 - \alpha)B^2}{2\eta} \quad (39)$$

$$C_1^1 = \frac{(1 - \alpha)C^1}{2\eta}, C_2^1 = \frac{(1 - \alpha)C^2}{2\eta} \quad (40)$$

$$D_1^1 = \frac{(1 - \alpha)D^1}{2\eta}, D_2^1 = \frac{(1 - \alpha)D^2}{2\eta} \quad (41)$$

and the controller parameters can be chosen to make  $\dot{V}(t)$  more negative in equation (36). Thus:

$$\frac{\partial E}{\partial \sigma_1^{ji}} = \max(A_1^1, A_2^1) \quad (42)$$

$$\frac{\partial E}{\partial \sigma_2^{ji}} = \max(B_1^1, B_2^1) \tag{43}$$

$$\frac{\partial E}{\partial m^{ji}} = \max(C_1^1, C_2^1) \tag{44}$$

$$\frac{\partial E}{\partial c^j} = \max(D_1^1, D_2^1) \tag{45}$$

As evident, the time delay effect is also present in the update law (equation 37), which enhances the efficiency of the proposed controller.

#### 4. SIMULATION

In this section, to demonstrate the effectiveness of the proposed method, the introduced controller is applied to control an inverted pendulum system mounted on a cart. The equation of this system in the disturbance-free state is given by [29]:

$$\begin{cases} \dot{x}_1 = x_2 \\ x_2 = \frac{g \sin(x_1) - m l x_2^2 \cos(x_1) \sin(x_1)}{l(-\frac{4}{3} m a \cos^2 x_1)} + \frac{a \cos(x_1)}{l(-\frac{4}{3} m a \cos^2 x_1)} u(t - \tau(t)) \end{cases} \tag{46}$$

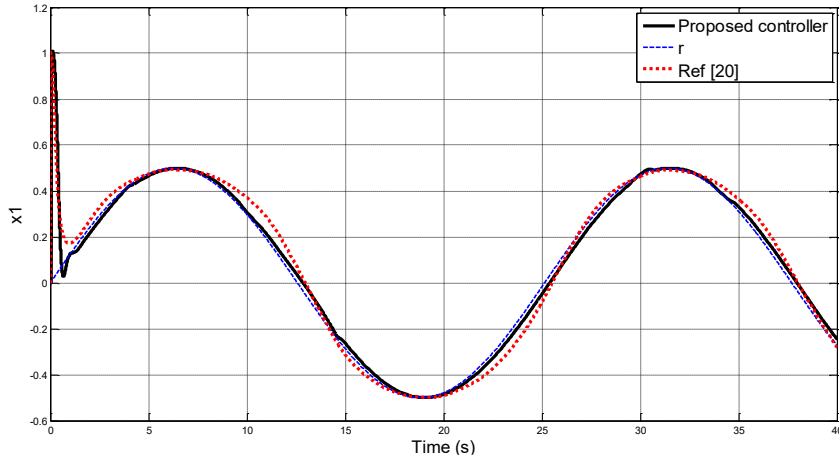
where  $x_1$  represents the pendulum angle,  $x_2$  represents its angular velocity,  $m=0.1$ kg is the weight of the pendulum, and  $mc=1$  kg is the weight of the cart. Also,  $a=1/(m+m_c)$  and  $g=9.8m/s^2$  are present. In this system, the control input  $u$ , representing the force to be applied to the cart to keep the pendulum in equilibrium or to track a specific angle, is determined by the network. Thus, the presence of the network can introduce time-varying delays in the control input [29]. For the Type-2 Neuro-Fuzzy Controller, three membership functions are used for each input (all three membership functions are Type-2), and the consequent part is assumed to be a constant. Initial values for the consequent part are chosen randomly, and the mean values of the centers for the Gaussian membership functions in the antecedent part are considered in the range of  $[-1, 1]$ . In this structure, the PD controller parameters are set to  $k_p = 10$  and  $k_d = 2$  to ensure closed-loop stability. The proposed method is compared with the controller introduced in reference [29], where two Type-1 Fuzzy Systems are used to approximate the functions  $g$  and  $f$ . It is worth noting that in the mentioned reference, the delay in the control input is approximated by Pade approximation as an additional state variable. This comparison aims to demonstrate the efficiency of the introduced controller in the presence of varying time delays and signal-to-noise ratios (SNR), which represent the noise power in the output sensor and were neglected in reference [29]. For the inverted pendulum system, consider the time-varying delay equation (in milliseconds):

$$\tau(t) = 1.5 + 0.5 \sin(3t) \tag{47}$$

Also, it is assumed that the measured output (pendulum angle) is accompanied by sensor noise  $n(t)$  expressed as:

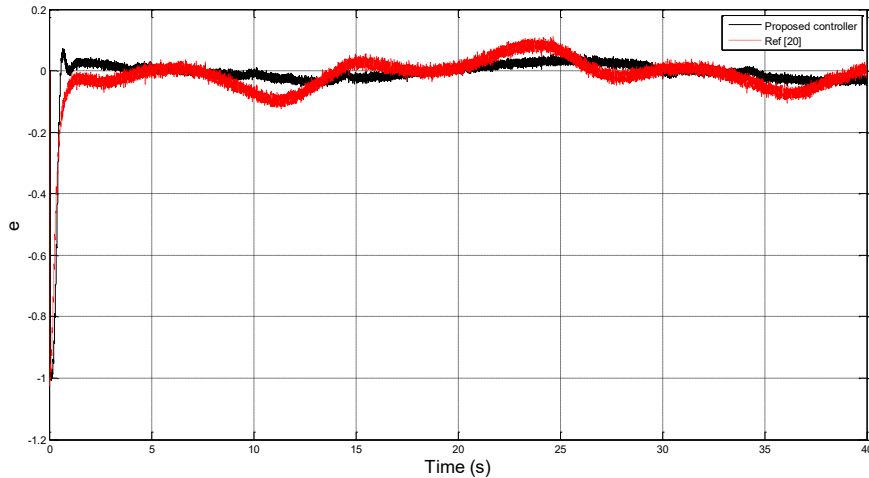
$$y'(t) = x_1(t) + n(t) \tag{48}$$

where  $n(t)$  is Gaussian noise with normal distribution. The initial state of the system is  $x_1=1$  and  $x_2=0$ , and the disturbance is considered as  $d(t) = 0.1 \sin(t/4)$ . The system's output for the mentioned controllers in response to a sinusoidal reference input is shown in Figure 3. In this case SNR=30db.



**Fig. 3.** System Output for Type-2 Neuro-Fuzzy Controllers and Reference [29]

According to Figure 3, both designed controllers track the reference input in the presence of disturbances and variable time delays. However, as demonstrated, the introduced Type-2 Neuro-Fuzzy Controller tracks the sinusoidal reference input with higher accuracy. The utilization of Type-2 fuzzy sets in the antecedent part, which better handles measurement noise and enhances the controller’s degrees of freedom, contributes to the superior performance of this controller. Additionally, Figure 4 shows the tracking error for both mentioned controllers, where it is evident that the tracking error amplitude for the proposed controller is lower compared to the other controller.



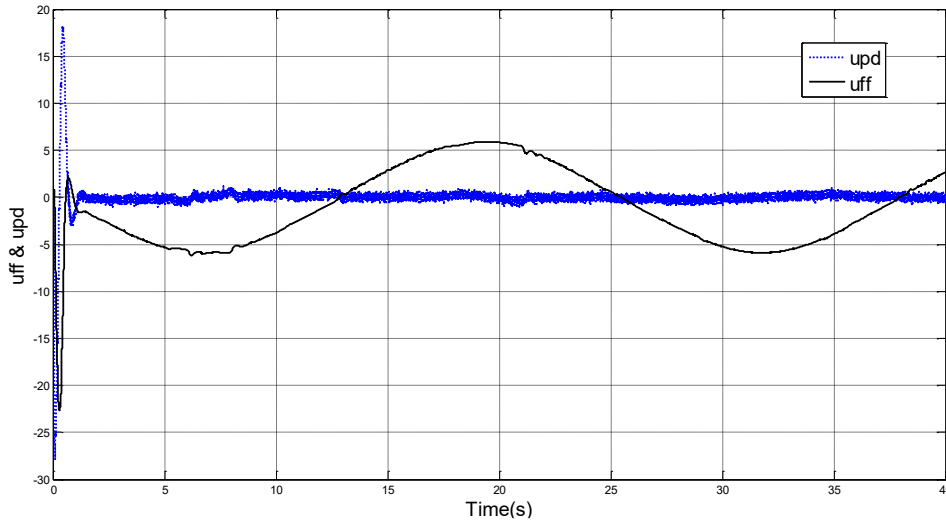
**Fig. 4.** Tracking Error for the Proposed Controller and Reference [29]

To demonstrate the effectiveness of the introduced Type-2 Neuro-Fuzzy Controller in handling measurement noise, simulations were conducted for various SNR values, and the Mean Squared Error (MSE) is summarized in Table 1.

**Table 1.** Mean Squared Error for Different Measurement Noise Levels

Signal-to-noise ratio (SNR)	Proposed controller	Controller proposed in [29]
80 db	0.025	0.035
30 db	0.0066	0.0093
20 db	0.0071	0.023

The table indicates that for high measurement noise levels, the performance difference between the introduced Type-2 Neuro-Fuzzy Controller and the one in reference [29] is significant, with the Type-2 Neuro-Fuzzy Controller exhibiting better noise resilience. In other words, when the measurement noise power is low, the performance difference between the two controllers is minimal. Additionally, Figure 5 illustrates the output of the Type-2 Neuro-Fuzzy Controller and the PD Controller.



**Fig. 5.** Output of the Type-2 Neuro-Fuzzy Controller (uff) and the Classical Controller (uPD) in the Introduced Structure

Figure 5 shows that initially, the PD Controller is active and stabilizes the system while training the leading controller. As the leading controller is trained and the output of the PD Controller approaches zero, the Type-2 Neuro-Fuzzy Network takes over the system control. It is noteworthy that when the leading controller assumes its role as the main controller, the inverse system is constructed, and tracking of the reference input is achieved.

## 5. CONCLUSION

This paper presented a design for a Type-2 Neuro-Fuzzy Controller combined with a classical controller for a nonlinear system with input delays. In this structure, the output of the classical PD controller was used to train the parameters of the Type-2 Neuro-Fuzzy Controller positioned in the lead path. To analyze the closed-loop stability of the controlled system, an appropriate Lyapunov-Krasovskii function was employed. Additionally, this function was integrated with a gradient descent algorithm to derive the parameter tuning laws for the Type-2 Neuro-Fuzzy Controller. The proposed strategy was compared with an indirect controller using Type-1 Fuzzy Systems, demonstrating the effectiveness of the proposed method in controlling the delayed inverted pendulum system and handling measurement noise. The derived relations for tuning the antecedent and consequent parts of the Type-2 Neuro-Fuzzy Controller do not depend on the system's dynamic equations, which consequently reduces computational complexity for online applications.

## Transparency Statement

The data supporting this study are available upon reasonable request to the corresponding author, subject to ethical and confidentiality considerations.

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## **Declaration of Interest**

The authors declare that they have no competing interests.

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